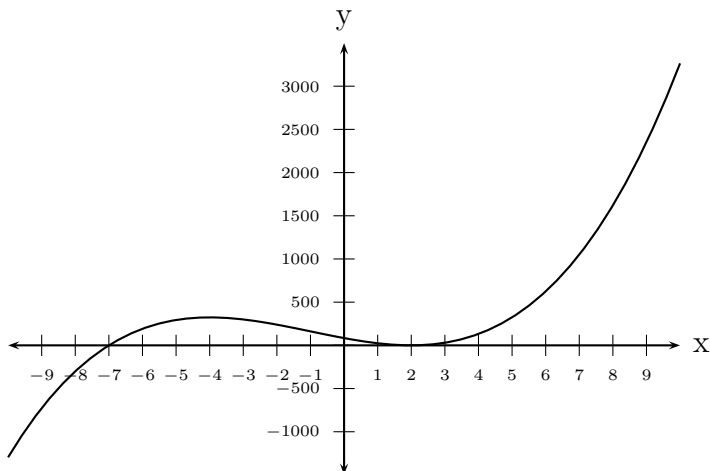# 7.1   Polynomials Practice Problems
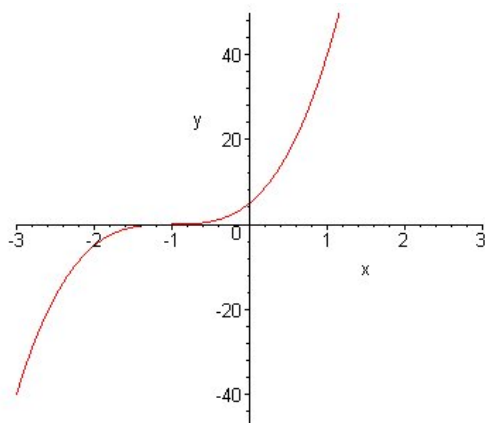
1. Let $f(x) = x^2$ and $g(x) = 4x^2 - 5x - 6$.

    - Describe the transformations that could be applied to the graph of f to obtain the graph of g.
    - Sketch the graph of g.
    - What is the vertex of the graph of g?
    - Does the graph of g have an absolute minimum or an absolute maximum? What is it?

2. Let $f(x) = x^2$ and $g(x) = -2x^2 + 3x + 4$.

    - Describe the transformations that could be applied to the graph of f to obtain the graph of g.
    - Sketch the graph of g.
    - What is the vertex of the graph of g?
    - Does the graph of g have an absolute minimum or an absolute maximum? What is it?

3. Find the quadratic function $f(x) = ax^2 + bx + c$ whose vertex is $(2, 3)$ and goes through the point $(-6, 4)$.

4. (Exercise 55 from Section 4.1 of your textbook) A field bounded on one side by a river is to be fenced on three sides so as to form a rectangular enclosure. If 200 feet of fencing is to be used, what dimensions will yield an enclosure of the largest possible area?

5. (Exercise 56 from Section 4.1 of your textbook) A rectangular box (with top) has a square base. The sum of the lengths of its 12 edges is 8 feet. What dimensions should the box have so that its surface area is as large as possible.

6. (Exercise 61 from Section 4.1 of your textbook) A potter can sell 120 bowls per week at $5 per bowl. For each 50 cent decrease in price, 20 more bowls are sold. What price should be charged to maximize sales income.

7. Describe the end behavior of each polynomial. Use correct mathematical symbols.

    (a) $P(x) = 2x^5 - 3x^2 + 76$
    (b) $R(x) = (2x + 3)^4(50 - x)^{100}$ (**HINT** What is the leading term?)
    (c) $S(x) = (1 - 2x)^{11}(x + 5)^4$

8. The graph of a polynomial $P(x)$ is shown below.

(a) Is the degree of the polynomial even or odd?

(b) Is the leading coefficient positive or negative?

(c) What can you say about the factors of this polynomial?

(d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = 24$

9. The graph shown below is NOT the graph of $y = h(x) = 5(x + 1)^4$. Which of the following are clues that this is NOT the graph of h?

(a) The graph crosses the x-axis at $(-1, 0)$, but it should not cross the x-axis at this point.

(b) The graph displays the wrong end behavior.

(c) The graph has the wrong x-intercepts.

(d) The graph does not have the right number of local extreme points to be the graph of a polynomial of degree 4.

2

10. Find the quotient and the remainder for each of the following division problems.

(a) $\dfrac{x^3 - 2x^2 + 7x + 1}{2x + 3}$

(b) $\dfrac{x^4 + 3x - 1}{x^2 + 2}$

(c) $\dfrac{2x + 3}{x^3 - 2x^2 + 7x + 1}$

11. Find the remainder of the following division problems.

(a) $\dfrac{x^{99} + 7}{x - 1}$

(b) $\dfrac{x^{100000} + x^{99999} + 4}{x + 1}$

(c) $\dfrac{x^{10} - 3}{x + 2}$

12. Find all the real roots of each polynomial.

(a) $P(x) = 2x^4 - 17x^3 + 31x^2 - 85x + 105$

(b) $Q(x) = 9x^4 - 30x^3 + 10x^2 + 16x - 9$