9 Functions Worksheet

1. Which of the following tables could describe a function? Explain your answer.

   (a)

Input	★	■	★	♦
Output	□	◦	△	△

   Not a function because the input ★ gives two different outputs.

   (b)

Input	♢	■	★	⋈
Output	♠	☺	♡	☺

   This is a function because each input gives exactly one output.

2. Which of the following equations define q as a function of r? Which of the following equations define r as a function of q?

   (a) $qr = 2$ **q is a function of r with implied domain $r \neq 0$ and r is a function of q with implied domain $q \neq 0$.**

   (b) $qr = 0$ **q is not a function of r because the input $r = 0$ results in many possible outputs. Similarly, r is not a function of q.**

   (c) $(q+1)^3 - r^2 = 7$ **q is a function of r, but r is not a function of q.**

3. Let $\text{Joni}(x) = x^2 + 1$.

   (a) What is $\text{Joni}(a+b)$? $\text{Joni}(\mathbf{a}+\mathbf{b}) = (\mathbf{a}+\mathbf{b})^{\mathbf{2}} + \mathbf{1}$

   (b) What is $\text{Joni}(x-5)$? $\text{Joni}(\mathbf{x}-\mathbf{5}) = (\mathbf{x}-\mathbf{5})^{\mathbf{2}} + \mathbf{1}$

4. Suppose $f(x) = x^2$ and $g(x) = (x+4)^2 - 1$. We can write g in terms of f as $g(x) = f(x+4) - 1$. For each of the following, write the given functions in terms of $f(x)$.

   (a) $a(x) = 2x^2$ $\mathbf{a}(\mathbf{x}) = \mathbf{2f}(\mathbf{x})$

   (b) $b(x) = (x-1)^2$ $\mathbf{b}(\mathbf{x}) = \mathbf{f}(\mathbf{x}-\mathbf{1})$

   (c) $e(x) = 3(x-2)^2 - 7$ $\mathbf{e}(\mathbf{x}) = \mathbf{3f}(\mathbf{x}-\mathbf{2}) - \mathbf{7}$

5. Let $g(x) = x^2 + x$.

   (a) What is $\dfrac{g(2x)}{2g(x)}$? $\frac{\mathbf{4x^2+2x}}{\mathbf{2x^2+2x}}$

   (b) What is $g(x^2)$? $\mathbf{x^4 + x^2}$

   (c) What is $(g(x))^2$? $\mathbf{(x^2+x)^2 = x^4 + 2x^3 + x^2}$

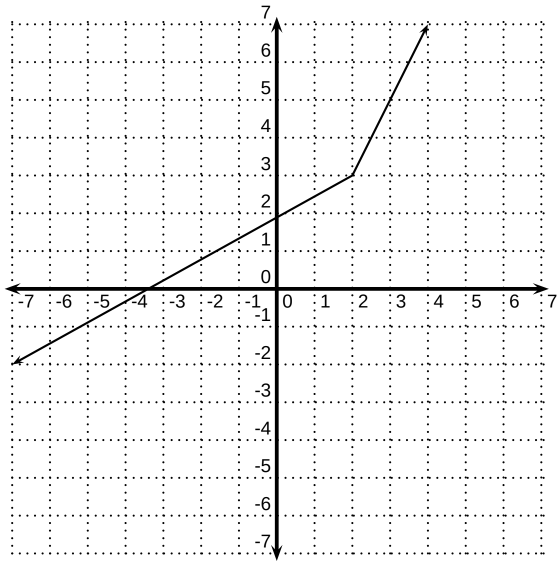
   (d) What is $\dfrac{g(x+h) - g(x)}{h}$? $\mathbf{2x + h + 1}$

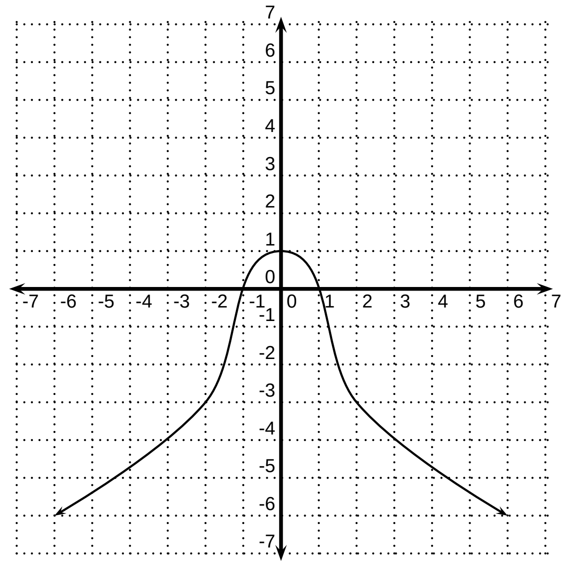
6. Let

$$h(x) = \begin{cases} 10 & \text{if } x < -4 \\ x^2 + 10 & \text{if } -4 \leq x \leq 6 \\ x + 15 & \text{if } x > 6 \end{cases}$$

- Find $h(5)$. **35**
- Find $h(-4)$. **26**
- Find $h(-6)$. **10**
- Find $h(6)$. **46**
- Find $h(10)$. **25**

7. For each of the graphs below, answer the following questions:
   - Is y a function of x?
   - Is x a function of y?

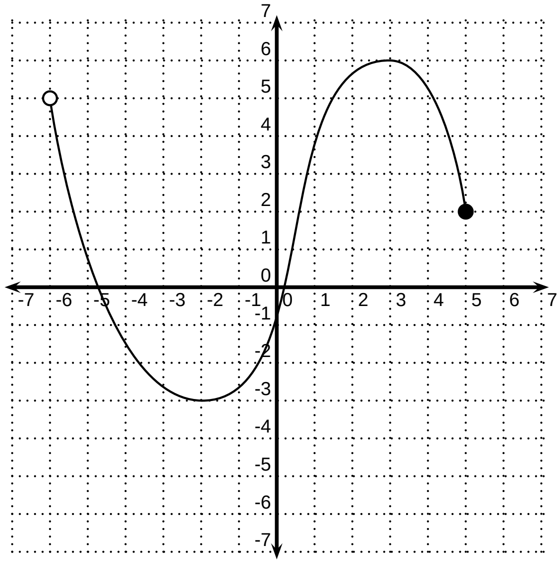


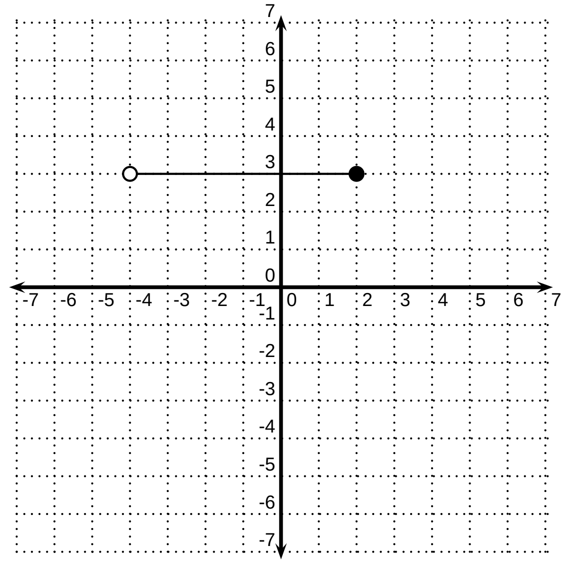


Yes for both questions. y **is a function of** x**;** x **is not a function of** y**.**

8. Find the domain of the following functions. Write the domain in interval notation.
   (a) $a(x) = x^5 + 2x^2 - 6$ $\quad (-\infty, \infty)$
   (b) $b(x) = \dfrac{32}{x^2 - 25}$ $\quad (-\infty, \mathbf{-5}) \cup (\mathbf{-5}, \mathbf{5}) \cup (\mathbf{5}, \infty)$
   (c) $c(x) = \sqrt{x+7}$ $\quad [\mathbf{-7}, \infty)$
   (d) $d(x) = \sqrt[3]{x+7}$ $\quad (-\infty, \infty)$
   (e) $3(x) = \dfrac{x+1}{x-5} + \dfrac{x+4}{2x+1}$ $\quad (-\infty, -\tfrac{\mathbf{1}}{\mathbf{2}}) \cup (-\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{5}) \cup (\mathbf{5}, \infty)$
   (f) $f(x) = \dfrac{1}{\sqrt[4]{10 - x}}$ $\quad (-\infty, \mathbf{10})$

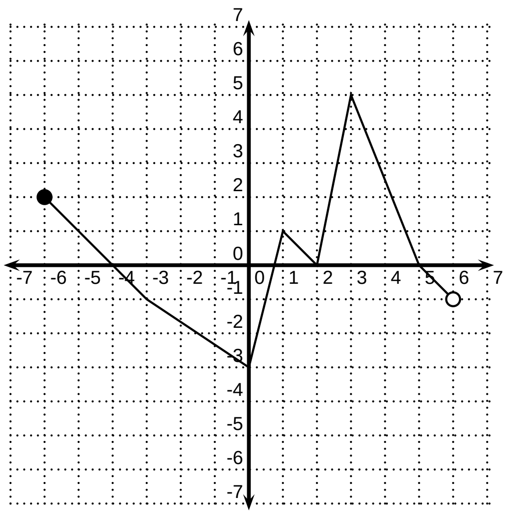
9. Find the domain and range of each of the following functions.





Domain: $(-\mathbf{6}, \mathbf{5}]$; **Range:** $[-\mathbf{3}, \mathbf{6}]$ **Domain:** $(-\mathbf{4}, \mathbf{2}]$; **Range:** $\{3\}$

10. The graph of $y = f(x)$ is shown below.



(a) For what x values is $f(x) \geq 0$? Write your answer in interval notation.
$[-\mathbf{6}, -\mathbf{4}] \cup [\mathbf{0.8}, \mathbf{5}]$

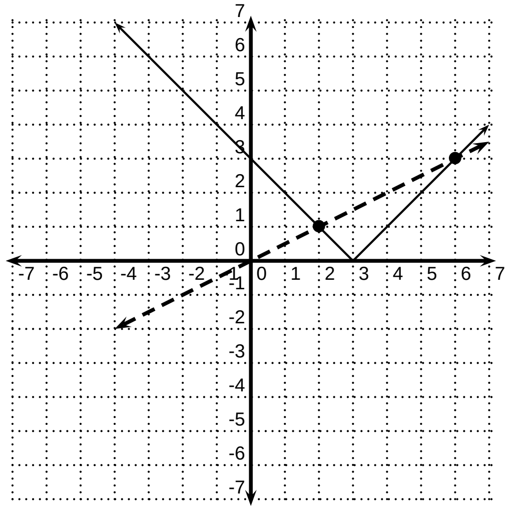
(b) For what x values is $f(x) < 0$? Write your answer in interval notation.
$(-\mathbf{0.4}, \mathbf{0.8}) \cup (\mathbf{5}, \mathbf{6})$

(c) For what x values is $f(x) \leq -1$? Write your answer in interval notation.
$[-\mathbf{3}, \mathbf{0.5}]$

(d) What is $\dfrac{f(3) - f(2)}{2f(-6)}$? $\mathbf{\frac{5}{4}}$

11. Let $f(x) = |x - 3|$. Find the average rate of change of $f(x)$ with respect to x as x changes from $x = 2$ to $x = 6$. Sketch a graph that illustrates the geometric interpretation of this average rate of change.

AROC $= \frac{1}{2}$ **(slope of the secant line)**



12. Bennett rides his bike from his apartment 3 miles to the park at an average speed of 9 miles per hour. He then realizes he forgot his baseball glove. He immediately turns and rides for 10 minutes to the sporting goods store, which is 2.5 miles away. What is Bennett's average speed from his apartment to the sporting goods store? **11 miles per hour**

13. Find $\frac{f(x+h)-f(x)}{h}$. (Assume $h \neq 0$.) Simplify your answer as much as possible.

    (a) $f(x) = 5x^2 - 2$
    $\mathbf{10x + 5h}$

    (b) $l(x) = 4x + 13$
    $\mathbf{4}$

    (c) $g(x) = x^2 - 3x$
    $\mathbf{2x + h - 3}$

14. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 2x - 15$.

    (a) Find $f(g(x))$. $\mathbf{f(g(x)) = \sqrt{x^2 + 2x - 15}}$

    (b) Find the domain $f(g(x))$. $(-\infty, \mathbf{-5}] \cup [\mathbf{3}, \infty)$

    (c) Find $g(f(x))$. $\mathbf{g(f(x)) = |x| + 2\sqrt{x} - 15}$

    (d) Find the domain $g(f(x))$. $[\mathbf{0}, \infty)$

15. Let $f(x) = x^2 + 3$ and $g(x) = 2 - x$.

    (a) Find $(f+g)(x)$. $\mathbf{(f+g)(x) = x^2 + 3 + 2 - x}$

    (b) Find $\frac{f}{g}(3)$. $\mathbf{-12}$

(c) Find $f(g(11))$. **84**

(d) Find $f(g(x))$. $\mathbf{f(g(x)) = (2-x)^2+3}$

(e) Find $g(f(2))$. $\mathbf{-5}$

(f) Find $g(f(x))$. $\mathbf{g(f(x)) = 2-(x^2+3)}$

(g) Find $f(f(x))$. $\mathbf{f(f(x)) = (x^2+3)^2+3}$

(h) Find $g(g(x))$. $\mathbf{g(g(x)) = 2-(2-x) = x}$

16. You have a 20% discount coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a 10% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Jamie. Jamie tells you that you can take advantage of both the 20% discount and the 10% discount. She will apply the cell provider discount and then apply the manufacturer discount to the reduced price. On your second trip, you talk to Mia. She also says that you can take advantage of both deals, but she tells you that she will apply the manufacturer discount and then apply the cell provider discount.

    Let x represent the original sticker price of the cell phone.

    (a) Suppose that only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the sticker price x. $\mathbf{f(x) = 0.80x}$

    (b) Suppose that only the 10% discount applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x. $\mathbf{g(x) = 0.90x}$

    (c) If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x)$, depending on the order in which the discounts are applied to the price. Find $f(g(x))$ and $g(f(x))$. $\mathbf{f(g(x)) = 0.72x}$ **and** $\mathbf{g(f(x)) = 0.72x}$

    (d) The price that Jamie is offering you is modeled by $\underline{\mathbf{f(g(x))}}$.

    (e) The price that Mia is offering you is modeled by $\underline{\mathbf{g(f(x))}}$.

17. Suppose that the graph of f contains the point $(-1, 5)$. Find a point that must be on the graph of g.

    (a) $g(x) = \dfrac{f(x)+1}{2}$ $\mathbf{(-1, 3)}$

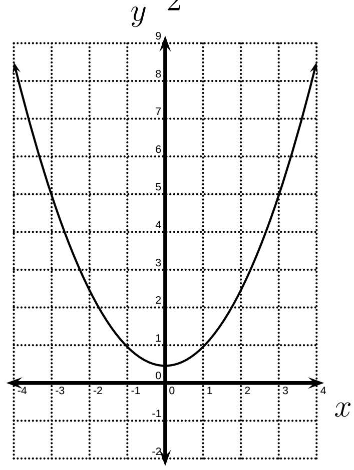
    (b) $g(x) = f(2x) - 3$ $\mathbf{(-\frac{1}{2}, 0)}$

    (c) $g(x) = f(4(x+1))$ $\mathbf{(-\frac{5}{4}, 5)}$

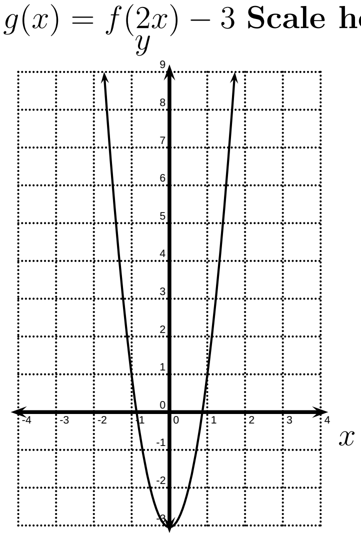
    (d) $g(x) = -f(x-5)$ $\mathbf{(4, -5)}$

18. Let $f(x) = x^2$. Explain how you would transform the graph of f to draw the graph of g. Sketch the graph of g.

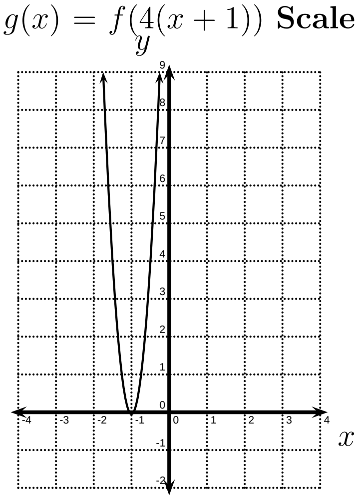
(a) $g(x) = \dfrac{f(x)+1}{2}$ **Shift up one unit. Scale vertically by a factor of $\frac{1}{2}$.**



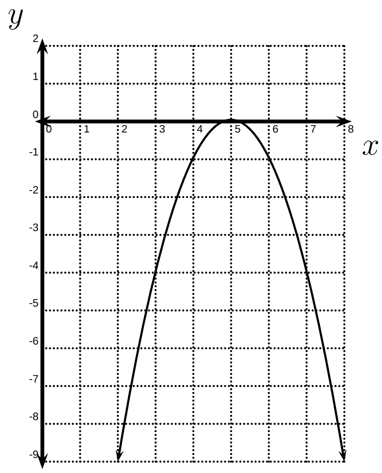
(b) $g(x) = f(2x) - 3$ **Scale horizontally by a factor of $\frac{1}{2}$. Shift down 3 units.**



(c) $g(x) = f(4(x+1))$ **Scale horizontally by a factor of $\frac{1}{4}$. Shift left 1 unit.**

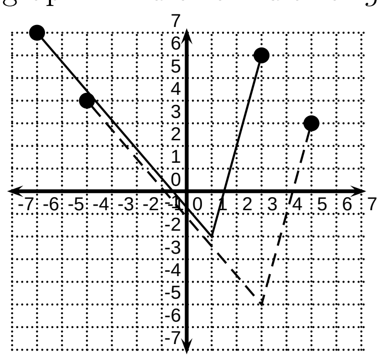


(d) $g(x) = -f(x-5)$ **Shift right 5 units. Reflect about the x-axis.**



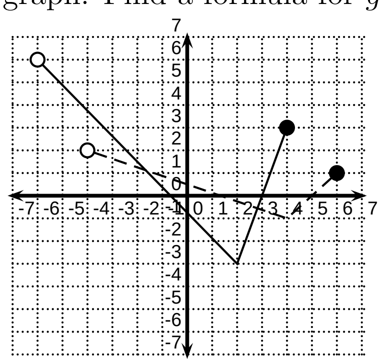
19. Suppose you want to transform the graph of $f(x)$ in the following way: First move every point to the right 2 units. Next scale vertically by a factor of 3. Finally, move down 7 units. Write a new function $g(x)$ in terms of $f(x)$ whose graph is this transformation.
$\mathbf{g(x) = 3f(x-2) - 7}$

20. The graph of $y = f(x)$ is the solid graph and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$ in terms of $f(x)$.



$\mathbf{g(x) = f(x-2) + 3}$

21. The graph of $y = f(x)$ is the solid graph and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$ in terms of $f(x)$.



$\mathbf{g(x) = \dfrac{f(x-2)}{3}}$

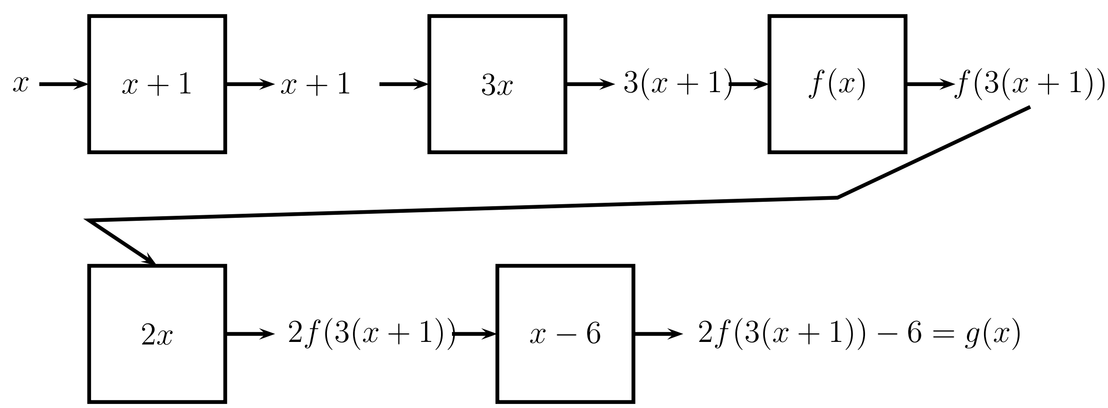
22. Write $h(x)$ as a composition of three simpler functions. (**HINT:** Think of placing x in a box. What happens first? second? etc.? There may be more than one correct answer.)

(a) $h(x) = \sqrt{x^3 + 5}$
$\mathbf{h(x) = k(g(f(x)))}$ where $\mathbf{f(x) = x^3, g(x) = x + 5, k(x) = \sqrt{x}}$

(b) $h(x) = \dfrac{3}{x^5 - 7}$
$\mathbf{h(x) = k(g(f(x)))}$ where $\mathbf{f(x) = x^5, g(x) = x - 7, k(x) = \dfrac{3}{x}}$

(c) $h(x) = 3(x + 5)^2$
$\mathbf{h(x) = k(g(f(x)))}$ where $\mathbf{f(x) = x + 5, g(x) = x^2, k(x) = 3x}$

23. Write $g(x) = 2f(3(x + 1)) - 6$ as a composition of five simpler functions (Hint:One of these functions should be f. Draw your composition as a machine diagram. Where is f in the machine order? How does this help illustrate how to transform the graph of $f(x)$ to draw the graph of $g(x)$?



24. Which of the following functions are one-to-one?

(a) The function that maps a word to the number of letters in the word.

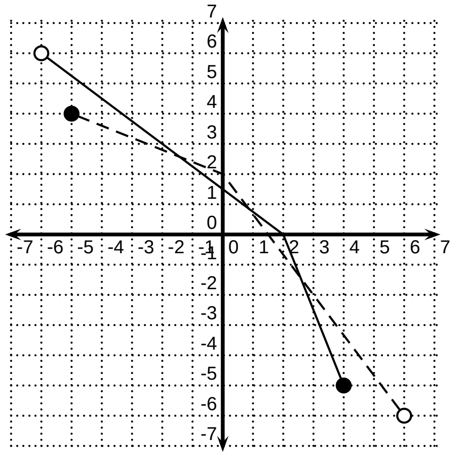
(b) The function that maps the year of a Summer Olymipcs to the winner of the marathon in that Olympics.

(c) The function that maps a U.S. state to its two letter postal code.

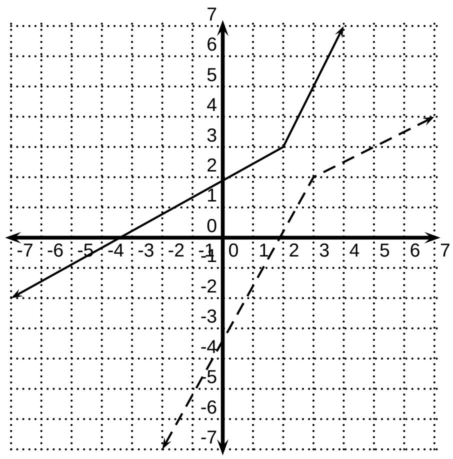
(d) The function that maps a person to his or her name.

(e) The function that maps a person to his or her address.

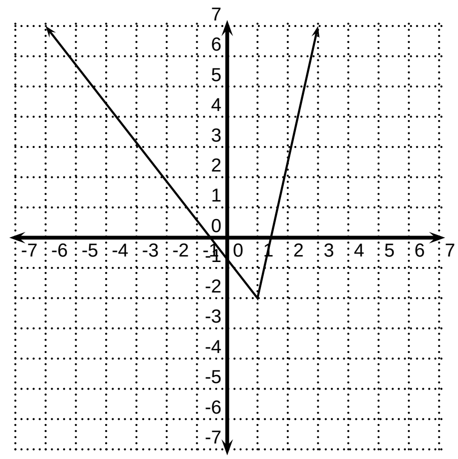
25. The graph of a one-to-one function is shown below. (How do you know that this is a one-to-one function?) Sketch the graph of its inverse.



26. Which of the following graphs display a one-to-one function? If the graph displays a one-to-one function, sketch its inverse.



NOT one-to-one



27. Find the inverse of the one-to-one functions below. Find the domains and ranges of the function and its inverse.

   (a) $f(x) = \dfrac{2 - x^3}{7}$

   $\mathbf{f^{-1}(x)} = \sqrt[3]{2 - 7x}$

   $\mathbf{f}$: Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$

   $\mathbf{f^{-1}}$: Domain $(-\infty, \infty)$, Range $(-\infty, \infty)$

   (b) $g(x) = \dfrac{x + 7}{x + 5}$

   $\mathbf{g^{-1}(x)} = \dfrac{\mathbf{7 - 5x}}{\mathbf{x - 1}}$

   $\mathbf{g}$: Domain $(-\infty, \mathbf{-5}) \cup (\mathbf{-5}, \infty)$, Range $(-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$

   $\mathbf{g^{-1}}$: Domain $(-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$, Range $(-\infty, \mathbf{-5}) \cup (\mathbf{-5}, \infty)$

   (c) $h(x) = \sqrt{x^5 - 2}$

   $\mathbf{h^{-1}(x)} = \sqrt[5]{x^2 + 2}$

$\mathbf{h}$: Domain $[\sqrt[5]{2}, \infty)$, Range $[\mathbf{0}, \infty)$
$\mathbf{h^{-1}}$: Domain $[\mathbf{0}, \infty)$, Range $[\sqrt[5]{2}, \infty)$

28. The following functions are not one-to-one. Restrict the domain so that the function is one-to-one. Remember that the range of the function with the restricted domain should be the same as the range of the original function. Find the inverse of the function with the restricted domain.

    (a) $f(x) = (x+2)^2$
    Restricted Domain: $[\mathbf{-2}, \infty)$
    $\mathbf{f^{-1}(x) = \sqrt{x} - 2}$

    (b) $g(x) = x^2 + 6x + 5$
    (Hint: $g(x) = (x+3)^2 - 4$)
    Restricted Domain: $[\mathbf{-3}, \infty)$
    $\mathbf{g^{-1}(x) = \sqrt{x+4} - 3}$

    (c) $h(x) = 2|x-5| - 3$
    Restricted Domain: $[\mathbf{5}, \infty)$
    $\mathbf{h^{-1}(x) = \frac{x+3}{2} + 5}$

29. Use the Round Trip Theorem to determine if the pair of functions are inverses of each other.

    (a) $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x}$
    yes because $\mathbf{f(g(x)) = x}$ and $\mathbf{g(f(x)) = x}$

    (b) $h(x) = \dfrac{3x+2}{7}$ and $j(x) = \dfrac{7x-2}{3}$
    yes because $\mathbf{h(j(x)) = x}$ and $\mathbf{j(h(x)) = x}$