7 A Strategy for Application Problems

Concepts:

- Solving Application Problems

(Section 2.3)

Although there is no hard and fast algorithm for solving application problems, there are strategies for approaching application problems that can help you to understand the problem, organize your thoughts, solve the problem, and communicate your solution.

A Strategy for Application Problems:

1. **READ**: Use a dictionary if necessary.
2. **DEFINE UNKNOWNS**: Use variables and describe them with words and/or a picture.
3. **DESCRIBE RELATIONSHIPS WITH MATHEMATICAL SYMBOLS**
4. **SIMPLIFY TO ONE EQUATION**
5. **SOLVE THE EQUATION**
6. **ANSWER THE QUESTION**: The answer may not be the variable you solved for.

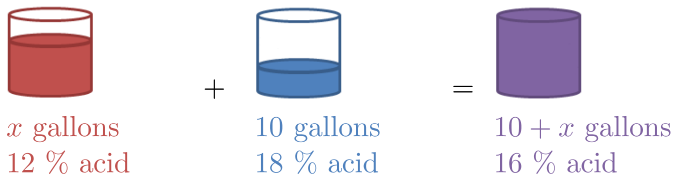
Example 7.1 (Number 1 from Section 2.3 in your textbook)
A student has exam scores of 88, 62, and 79. What score does he need on the fourth exam to have an average of 80?

Let x be the student's fourth exam score. We want to find x such that

$$\begin{aligned} \frac{88+62+79+x}{4} &= 80 \\ \frac{229+x}{4} &= 80 \\ 229+x &= 320 \\ x &= 91 \end{aligned}$$

So, the student needs a 91 on the fourth exam to have a test average of 80.

Example 7.2 (Similar to Number 2 from Section 2.3 in your textbook)
How many gallons of a 12% acid solution should be combined with 10 gallons of an 18% acid solution to obtain a 16% acid solution?



Note:
$$\text{concentration} = \frac{\text{gallons of pure acid}}{\text{total gallons of solution}}.$$
Rearranging, we have,
$$\text{gallons of pure acid} = (\text{concentration})(\text{total gallons of solution})$$

Since acid is neither created nor destroyed, we have the equation:

$$\text{gallons of acid in 12\% solution} + \text{gallons of acid in 18\% solution} = \text{gallons of acid in 16\% solution}$$

That is,
$$\begin{aligned}(.12)x + (.18)(10) &= .16(10 + x)\\ .12x + 1.8 &= 1.6 + .16x\\ 0.2 &= 0.04x\\ 5 &= x\end{aligned}$$

So, 5 gallons of 12% acid should be added.

Example 7.3 (Example 8 from Section 2.3 in your textbook)
A pilot wants to make the 840-mile round trip from Cleveland to Peoria and back in 5 hours flying time. Going to Peoria, there will be a headwind of 30 mph, that is, a wind opposite to the direction the plane is flying. It is estimated that on the return trip to Cleveland, there will be a 40-mph tailwind (in the direction the plane is flying). At what constant speed should the plane be flown?

Let x be the plane's constant speed. (This is the work that the plane's engine does.) The wind may affect the plane's actual speed. On the trip from Cleveland to Peoria, the plane travels at $x - 30$ mph and on the trip from Peoria to Cleveland, the plane travels at $x + 40$. Recall

$$\text{distance} = \text{rate} \cdot \text{time} \qquad \text{or,} \qquad \text{time} = \frac{\text{distance}}{\text{rate}}$$

Since the journey should take 5 hours, the we have the equation

$$\begin{aligned}\begin{matrix}\text{time to fly from}\\ \text{Cleveland to Peoria}\end{matrix} + \begin{matrix}\text{time to fly from}\\ \text{Peoria to Cleveland}\end{matrix} &= 5\\ \frac{420}{x-30} + \frac{420}{x+40} &= 5\\ 420(x+40) + 420(x-30) &= 5(x+40)(x-30)\\ 420x + 16800 + 420 - 12600 &= 5x^2 + 50x - 6000\\ 0 &= 5x^2 - 790x - 10200\\ x &= \frac{790 \pm \sqrt{790^2 - 4(5)(-10200)}}{2(5)}\\ &= 170, -12\end{aligned}$$

Since x is the plane's constant speed, and therefore should be a positive number, we conclude the plane's constant speed is approximately 170 mph.

Example 7.4 (Number 7 from Section 2.3 of your textbook)
The diameter of a circle is 16 cm. By what amount must the radius be decreased to decrease the area by 48π square centimeters?

The area of the circle is

$$\pi r^2 = \pi \left(\frac{16}{2}\right)^2 = \pi(8)^2 = 64\pi$$

If we decrease the area by 48π, then the area of the new circle is

$$64\pi - 48\pi = 16\pi = \pi(4)^2 =$$

That is, our new circle will have a radius of 4 cm. Thus the radius decreased by $8 - 4 = 4$cm.

Example 7.5 (Number 6 from Section 2.3 in your textbook)
A merchant has 5 pounds of mixed nuts that cost \$30. He wants to add peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 50 pounds of a mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?

	Current Mix	Peanuts	Cashews	New Mix
Weight (lbs.)	5	p	c	50
Total Cost (\$)	30	$1.50p$	$4.50c$	$2.90(5) = 145$
Cost per pound (\$/lb.)	30/5=6	1.50	4.50	2.90

By comparing the amount (weight) of nuts used to create the new mix, we obtain

$$5 + p + c = 50$$

or,

$$p = 45 - c$$

By comparing the cost of each type of nut to create the new mix, we obtain

$$\begin{aligned}
\begin{pmatrix}\text{Total cost of}\\ \text{current mix}\end{pmatrix} + \begin{pmatrix}\text{Total cost}\\ \text{of peanuts}\end{pmatrix} + \begin{pmatrix}\text{Total cost}\\ \text{of cashews}\end{pmatrix} &= \begin{pmatrix}\text{Total cost of}\\ \text{new mix}\end{pmatrix} \\
145 &= 30 + 1.5p + 4.5c \\
145 &= 30 + 1.5(45 - c) + 4.5c \\
145 &= 30 + 67.5 - 1.5c + 4.5c \\
47.5 &= 3c \\
15.83 &\approx c
\end{aligned}$$

That is, we need approximately 15.83 pounds of cashews. Substituting, we find

$$p = 45 - c \approx 45 - 15.83 \approx 29.17$$

So, we need approximately 29.17 pounds of peanuts.