MA109 — College Algebra Spring 2017
Final Exam 2017-05-02

Name: Key **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	12.	**a**	b	c	d	e
2.	a	b	**c**	d	e	13.	a	b	c	**d**	e
3.	a	b	c	**d**	e	14.	a	**b**	c	d	e
4.	a	b	c	d	**e**	15.	**a**	b	c	d	e
5.	a	b	c	d	**e**	16.	a	**b**	c	d	e
6.	a	b	**c**	d	e	17.	a	b	c	d	**e**
7.	a	b	**c**	d	e	18.	a	b	c	**d**	e
8.	a	b	c	**d**	e	19.	a	b	**c**	d	e
9.	a	b	c	d	**e**	20.	a	**b**	c	d	e
10.	a	b	c	**d**	e	21.	a	b	**c**	d	e
11.	a	**b**	c	d	e	22.	a	**b**	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: KEY

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve the equation for x.

$$\log(4x+3) = \log(6x+2) + \log(10)$$

$\log(4x+3) = \log[(6x+2)\cdot 10]$

$\log(4x+3) = \log(60x+20)$

$4x+3 = 60x+20$

$3 = 56x+20$

$-17 = 56x$

$\frac{-17}{56} = x$

Possibilities:

(a) $x = -\frac{17}{56}$ only

(b) $x = \log\left(\frac{7}{4}\right)$ and $x = \log\left(\frac{4}{3}\right)$

(c) $x = \frac{-26 \pm \sqrt{580}}{48}$ only

(d) $x = \frac{7}{4}$ and $x = \frac{4}{3}$

(e) No solution

2. How much money should be invested at 6.25% interest, compounded monthly, so that 11 years later the investment will be worth \$10,000?

$P(t) = P_0\left(1+\frac{r}{n}\right)^{n\cdot t}$

$10{,}000 = P_0\left(1+\frac{.0625}{12}\right)^{(12)(11)}$

$10{,}000 = P_0\left(1+\frac{.0625}{12}\right)^{132}$

$\frac{10{,}000}{\left(1+\frac{.0625}{12}\right)^{132}} = P_0$

$5037.30 \approx P_0$

Possibilities:

(a) \$19,481.32

(b) \$909.09

(c) \$5,037.30

(d) \$5,925.93

(e) \$5,133.12

3. You have already invested \$400 in a stock with an annual return of 10%. How much of an additional \$1,200 should be invested at 20% and how much at 5% so that the total return on the entire \$1,600 is 15%?

The multiple choice problem only asks for the amount at 20%.

x = amount @ 20%
y = amount @ 5%

Possibilities:

(a) \$533.33 at 20%

(b) \$800 at 20%

(c) \$266.67 at 20%

(d) \$933.33 at 20%

(e) \$1,066.67 at 20%

Amount $\Rightarrow x + y + 400 = 1600$

Value $\Rightarrow .20x + .05y + (.10)(400) = (.15)(1600)$

$y = 1600 - 400 - x = \boxed{1200 - x}$

$.20x + .05(1200 - x) + (.10)(400) = (.15)(1600)$

$.20x + 60 - .05x + 40 = 240$

$.15x + 100 = 240$

$.15x = 140$

$x = \frac{140}{.15} \approx \$933.33$

4. The radioactive element Fluorine-18 has a half-life of 110 minutes and is used as a medical radiotracer in PET scans. How long should it take for 81 milligrams to decay to 12 milligrams?

Possibilities:

(a) About 2.75 minutes

(b) About 0.03 minutes

(c) About 742.50 minutes

(d) About 7590.00 minutes

(e) About 303.04 minutes

$P(t) = 81(.5^{\frac{t}{110}})$

$12 = 81(.5^{\frac{t}{110}})$

$\frac{12}{81} = .5^{\frac{t}{110}}$

$\ln(\frac{12}{81}) = \ln(.5^{\frac{t}{110}})$

$\ln(\frac{12}{81}) = \frac{t}{110}\ln(.5)$

$110\ln(\frac{12}{81}) = t\ln(.5)$

$\frac{110\ln(\frac{12}{81})}{\ln(.5)} = t \approx \boxed{303.04 \text{ minutes}}$

$P(t) = P_0 a^t$

$P(110) = P_0 a^{110}$

$.5P_0 = P_0 a^{110}$

$(.5)^{1/110} = a$

5. Solve: $2048^{19x-4} = 64$

Possibilities:

(a) $\sqrt{-4} - \sqrt{19}$

(b) $\sqrt[2048]{-4} - \sqrt[64]{19}$

(c) $\frac{64}{19}$

(d) $\log(-4) - \log(19)$

(e) $\frac{50}{209}$

$2048^{19x-4} = 64$

$(2^{11})^{19x-4} = 2^6$

$2^{11(19x-4)} = 2^6$

$11(19x-4) = 6$

$209x - 44 = 6$

$209x = 50$

$x = 50/209$

6. Write the domain of the function $h(x) = \log(12 - x)$ in interval notation.

Possibilities:

(a) $(-\infty, 12) \cup (12, \infty)$

(b) $(-\infty, \infty)$

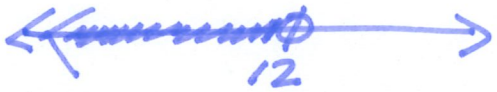
(c) $(-\infty, 12)$

(d) $(-\infty, -12]$

(e) $(-12, \infty)$

$12 - x > 0$

$12 > x$



$(-\infty, 12)$

7. Let $f(x) = 9^x$. Which of the following is $f(-2)$?

Possibilities:

(a) $-\frac{1}{3}$

(b) $\frac{1}{9}$

(c) $\frac{1}{81}$

(d) 81

(e) 3

$f(x) = 9^x$

$f(-2) = 9^{-2}$

$f(-2) = \frac{1}{9^2}$

$f(-2) = \frac{1}{81}$

8. Determine the end behavior of the following function.

$$(7-5x^2)^{11}$$

Possibilities:

(a) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(b) $y \to 0$ as $x \to \infty$ and $y \to 0$ as $x \to -\infty$

(c) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(d) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(e) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

* leading term

$(-5x^2)^{11}$

$(-5)^{11} x^{22}$

$-5^{11} x^{22}$

negative, even

$y \to -\infty$ as $x \to -\infty$; $y \to -\infty$ as $x \to \infty$

9. Determine the end behavior of the following function.

$$\frac{(5x^2+3)(9x+13)}{11x^3+7}$$

Possibilities:

(a) $y \to +\infty$ as $x \to \infty$ and $y \to +\infty$ as $x \to -\infty$

(b) $y \to +\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(c) $y \to \frac{225}{11}$ as $x \to \infty$ and $y \to \frac{225}{11}$ as $x \to -\infty$

(d) $y \to 0$ as $x \to \infty$ and $y \to 0$ as $x \to -\infty$

(e) $y \to \frac{45}{11}$ as $x \to \infty$ and $y \to \frac{45}{11}$ as $x \to -\infty$

* ratio of leading terms

$\frac{(5x^2)(9x)}{11x^3} = \frac{45x^3}{11x^3} = \frac{45}{11}$

$y \to \frac{45}{11}$ as $x \to \infty$

$y \to \frac{45}{11}$ as $x \to -\infty$

10. Let $r(x) = \dfrac{3x-5}{x-11}$. Find the asymptotes of r.

HA: ratio of lead. terms $\Rightarrow \frac{3x}{x} = 3$

So, $y = 3$ is horizontal asymptote.

Possibilities:

(a) The vertical asymptote is $x = 11$ and the horizontal asymptote is $y = 1$.

(b) The vertical asymptote is $x = 5$ and the horizontal asymptote is $y = 3$.

(c) The vertical asymptote is $x = 5$ and the horizontal asymptote is $y = 11$.

(d) The vertical asymptote is $x = 11$ and the horizontal asymptote is $y = 3$.

(e) The vertical asymptote is $x = \frac{5}{3}$ and the horizontal asymptote is $y = 11$.

VA: 0's in denominator NOT 0's in numerator

$x - 11 = 0$

$x = 11$ is vertical asymptote

11. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -2 \\ x^2 + 3 & \text{if } -2 < x \leq 5 \\ -2x - 5 & \text{if } x > 5 \end{cases}$$

Since $-2 < 4 \leq 5$

Find $f(4)$.

Possibilities:

(a) 11

(b) 19

(c) 209

(d) -13

(e) 4

$f(4) = (4)^2 + 3$

$= 16 + 3$

$= 19$

12. Solve for z.

$3z^2 - 11z + 2 = 0$

Possibilities:

(a) $\frac{11 \pm \sqrt{97}}{6}$

(b) $\frac{-11 \pm \sqrt{97}}{6}$

(c) $\frac{11}{6} \pm \sqrt{115}$

(d) $\frac{11 \pm \sqrt{145}}{6}$

(e) $\frac{-11 \pm \sqrt{145}}{6}$

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(2)}}{2(3)}$

$= \frac{11 \pm \sqrt{121 - 24}}{6}$

$= \frac{11 \pm \sqrt{97}}{6}$

13. Find an equation for the line through the points $(4, 3)$ and $(6, 2)$.

Possibilities:

(a) $y + 3 = -\frac{1}{2}(x + 4)$

(b) $y + 3 = -2(x + 4)$

(c) $y - 3 = -2(x - 4)$

(d) $y - 3 = -\frac{1}{2}(x - 4)$

(e) $y = 2(x - 4) - 3$

x_1, y_1 x_2, y_2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{6 - 4} = \frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{2}(x - 4)$

14. Which of the following statements best describes the system of equations?

$$\begin{cases} x + y = 4 \\ 2x + 2y = 8 \end{cases}$$

$1+3 \stackrel{?}{=} 4$, $4 \checkmark 4$; $2+2 \stackrel{?}{=} 4$, $4 \checkmark 4$; $2(1)+2(3) \stackrel{?}{=} 8$, $2+6 \checkmark 8$; $2(2)+2(2) \stackrel{?}{=} 8$, $4+4 \checkmark 8$; $0+0 \stackrel{?}{=} 4$, $0 \neq 4$

(-2): $-2x-2y=-8$, $2x+2y=8$, $0=0$ TRUE!

Possibilities:

(a) The system is consistent. It has exactly one solution which is $(4, 8)$.

(b) The system is dependent. Two solutions to the system are $(1, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(0, 0)$.

(c) The system is dependent. Two solutions to the system are $(4, 8)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$.

(d) The system is inconsistent. Therefore the system has no solutions.

(e) The system is dependent. Every point is a solution to the system.

coinciding lines $\Rightarrow$ DEPENDENT System

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15. Let $f(x) = 4x^2 + 3x - 16$. Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $8x + 3 + 4h$

(b) $\dfrac{6xh + 3h^2 - 2}{h}$

(c) $8x + 4h$

(d) $4h$

(e) 1

$f(x+h) = 4(x+h)^2 + 3(x+h) - 16 = 4(x^2 + 2xh + h^2) + 3x + 3h - 16$

$= 4x^2 + 8xh + 4h^2 + 3x + 3h - 16$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{(4x^2 + 8xh + 4h^2 + 3x + 3h - 16) - (4x^2 + 3x - 16)}{h}$

$= \dfrac{8xh + 4h^2 + 3h}{h} = \dfrac{h(8x + 4h + 3)}{h}$

$= \boxed{8x + 4h + 3}$

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16. The point $(4, 3)$ is on the graph of which of the following equations?

Possibilities:

(a) $4x + 12 = 4y + 12$

(b) $4x + 12 = xy + 16$

(c) $xy = 0$

(d) $x = y - 1$

(e) $xy + 16 = xy + 12$

a) $4(4)+12 \stackrel{?}{=} 4(3)+12 \rightarrow 16+12 \neq 12+12$

b) $4(4)+12 \stackrel{?}{=} (4)(3)+16 \rightarrow 16+12 \checkmark 12+16$

c) $(4)(3) \stackrel{?}{=} 0 \rightarrow 12 \neq 0$

d) $4 \stackrel{?}{=} 3-1 \rightarrow 4 \neq 2$

e) $(4)(3)+16 \stackrel{?}{=} (4)(3)+12 \rightarrow 12+16 \neq 12+12$

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17. Solve for a in $\dfrac{(6a-7)^3}{2} = 4$.

Possibilities:

(a) $\dfrac{29}{18}$

(b) $\dfrac{2}{3}$

(c) $7 \pm \sqrt{8}/6$

(d) $\dfrac{4913}{2}$

(e) $\dfrac{3}{2}$

$$2\left(\frac{(6a-7)^3}{2}\right) = (4)\,2$$

$$(6a-7)^3 = 8$$

$$6a - 7 = \sqrt[3]{8}$$

$$6a - 7 = 2$$

$$6a = 9$$

$$a = \frac{9}{6} = \frac{3}{2}$$

18. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = p(1200 - 100p).$$

What per-ticket price maximizes the teams total revenue?

Possibilities:

(a) \$12.00

(b) \$0.04

(c) \$49.83

(d) \$6.00

(e) \$3600.00

$$R(p) = 1200p - 100p^2$$

$$= -100p^2 + 1200p$$

"a" "b"

Vertex (h, k)

$\left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right)\right)$

$-\frac{b}{2a}$: # of tickets to get MAX Revenue

$R\left(-\frac{b}{2a}\right)$: MAX revenue

$$h = -\frac{b}{2a} = \frac{-1200}{2(-100)} = \frac{-1200}{-200} = \$6.00$$

19. Explain how the graph of $g(x) = \sqrt{x-2} + 4$ is obtained from the graph of $f(x) = \sqrt{x}$.

Possibilities:

(a) Shift the graph of f left 4 units and shift down 2 units to obtain the graph of g.

(b) Shift the graph of f right 4 units and shift up 2 units to obtain the graph of g.

(c) Shift the graph of f right 2 units and shift up 4 units to obtain the graph of g.

(d) Shift the graph of f right 2 units and shift down 4 units to obtain the graph of g.

(e) Shift the graph of f left 2 units and shift up 4 units to obtain the graph of g.

right 2, up 4

$f(x) = \sqrt{x}$

$f(x-2) = \sqrt{x-2}$

$f(x-2) + 4 = \sqrt{x-2} + 4$

$f(x-2) + 4 = g(x)$

right 2 units, up 4 units

20. The equation of a parabola was printed below, but ink spilled on some of the numbers. Try to answer the question anyways:

$$y = \blacksquare\,(x-4)^2 + 17$$

Does this parabola open up or down? What is its vertex?

$y = (?)(x-4)^2 + 17$

$y = a(x-h)^2 + k$

vertex $\rightarrow (h,k) \Rightarrow (4,17)$

$a<0$ opens down
$a>0$ opens up
} ?

Possibilities:

(a) The vertex is (17,4), but up/down can't be read

(b) The vertex is (4,17), but up/down can't be read

(c) Down, but the vertex can't be read

(d) Up, but the vertex can't be read

(e) The vertex is (17,-4), but up/down can't be read

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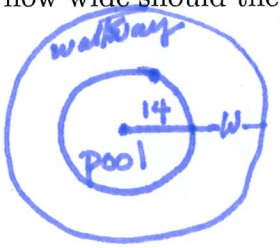
21. A concrete walk of uniform width is to be built around a giant circular pool. The radius of the pool is 14 meters, and enough concrete is available to cover 38.09π square meters (approximately). If all the concrete is to be used, how wide should the walk be (approximately)? Choose the closest answer.



Area Big circle − Area pool = Area walkway

$\pi(14+w)^2 - \pi(14)^2 = 38.09\pi$

$14^2 + 28w + w^2 - 14^2 = 38.09$

$w^2 + 28w - 38.09 = 0$

$w = \frac{-28 \pm \sqrt{28^2 - 4(1)(-38.09)}}{2(1)} = \frac{-28 \pm \sqrt{936.36}}{2}$

$w \approx 1.3$

Possibilities:

(a) 24.1 meters wide

(b) 14 meters wide

(c) 1.3 meters wide

(d) 7.83 meters wide

(e) 2.72 meters wide

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22. Find all distinct, real solutions x to $(x^2-2)(x-4)(x-9) = 0$.

$x^2 - 2 = 0$ $\quad x-4=0$ $\quad x-9=0$

$x^2 = 2$ $\quad x = 4$ $\quad x = 9$

$x = \pm\sqrt{2}$

Possibilities:

(a) $x = 2$, $x = 4$, and $x = 9$

(b) $x = \pm\sqrt{2}$, $x = 4$, and $x = 9$

(c) $x = \pm\sqrt{2}$, $x = -4$, and $x = -9$

(d) $x = -2$, $x = -4$, and $x = -9$

(e) No solution

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Exponential Growth Model If n_0 is the initial size of a population that experiences **exponential growth**, then the population $n(t)$ at time t increases according to the model:

$$\boxed{n(t) = n_0 e^{rt}}$$

where r is the relative rate of growth of the population (expressed as a proportion of the population).

Radioactive Decay Model: If m_0 is the initial mass of a radioactive substance with half-life h, then the mass $m(t)$ remaining at time t is modeled by the function:

$$\boxed{m(t) = m_0 e^{-rt}}$$

where $r = \dfrac{\ln 2}{h}$.

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$