

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

a b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

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For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the indicated value of the function when $x = \sqrt{7} + 6$.

$$f(x) = \sqrt{x+4} - x - 9$$

$$f(\sqrt{7} + 6) =$$

Possibilities:

- (a) $\sqrt{10} - 15$
 (b) $\sqrt{17} - \sqrt{7} - 15$
 (c) $\sqrt{\sqrt{7} + 10} - \sqrt{7} - 3$
 (d) -5
 (e) $\sqrt{\sqrt{7} + 10} - \sqrt{7} - 15$

$$\begin{aligned} f(\sqrt{7} + 6) &= \sqrt{(\sqrt{7} + 6) + 4} - (\sqrt{7} + 6) - 9 \\ &= \sqrt{\sqrt{7} + 6 + 4} - \sqrt{7} - 6 - 9 \\ &= \sqrt{\sqrt{7} + 10} - \sqrt{7} - 15 \end{aligned}$$

2. Find $f(2)$ if $f(x) = \begin{cases} 8 & \text{if } x \leq 1 \\ 2x + 6 & \text{if } 1 < x \leq 3 \\ 3x + 3 & \text{if } 3 < x \leq 5 \\ 18 & \text{if } x > 5 \end{cases}$ $\leftarrow 1 < \textcircled{2} \leq 3$

Possibilities:

- (a) 8
 (b) 10
 (c) 9
 (d) 12
 (e) 18

$$\begin{aligned} f(2) &= 2(2) + 6 \\ &= 4 + 6 \\ &= 10 \end{aligned}$$

3. Find the domain of $\sqrt{\frac{x-7}{5}}$ ← must be non-negative!

Possibilities:

(a) $(-\infty, 7) \cup (7, \infty)$

(b) $(7, \infty)$

(c) $(-\infty, \infty)$

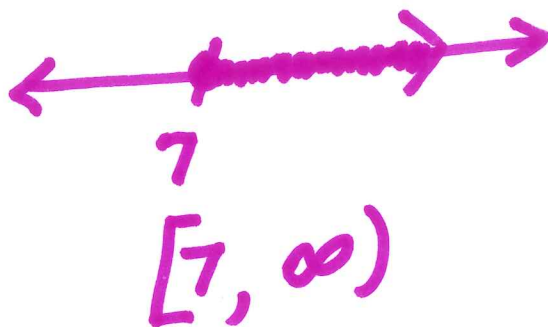
(d) $[7, \infty)$

(e) $[\frac{7}{5}, \infty)$

$$5 \left(\frac{x-7}{5} \right) \geq 0 (5)$$

$$x-7 \geq 0$$

$$x \geq 7$$



4. Find the domain of $\frac{7}{\sqrt[3]{x-5}}$ ← cannot equal 0!

Possibilities:

(a) $[\frac{7}{5}, \infty)$

(b) $(-\infty, \infty)$

(c) $(5, \infty)$

(d) $(-\infty, 5) \cup (5, \infty)$

(e) $[5, \infty)$

$$\left(\sqrt[3]{x-5} \right)^3 = (0)^3$$

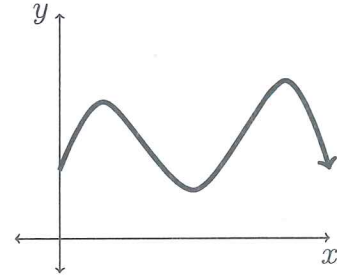
$$x-5 = 0$$

$$x = 5$$

Thus, $x \neq 5$

$$(-\infty, 5) \cup (5, \infty)$$





5. Which situation below is most reasonably depicted in this graph:

Possibilities:

- (a) y is the number of bacteria at time x if the bacteria experience a steady rate of exponential growth.
- (b) y is the temperature of left-over food at time x if the food is placed in the refrigerator at time $x = 0$.
- (c) y is the distance from home at time x as you run to the end of the block and back at a steady pace.
- (d) y is the amount of water in a bucket at time x if a hole is made in the bucket at time $x = 0$.
- (e) y is the outside temperature after x hours, if $x = 0$ is around midday on the first day.

a) increasing only ↗
 b) decreasing to refrigerator temp ↘
 c) out & back ↗ ↘
 d) decreasing to 0 ↘
 e) ONLY oscillating growth

6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 15 to 20 seconds.

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	50	200	450	800	1250	1800

Possibilities:

- (a) 30 feet per second
- (b) 60 feet per second
- (c) 40 feet per second
- (d) 90 feet per second
- (e) 70 feet per second

$$\begin{aligned}
 \text{Avg. speed} &= \frac{\Delta d}{\Delta t} \\
 &= \frac{800 - 450}{20 - 15} \\
 &= \frac{350 \text{ ft}}{5 \text{ sec}} \\
 &= 70 \text{ ft/sec}
 \end{aligned}$$

7. Simplify the formula for the average rate of change of $f(x) = (x - 7)^2 + 5$ from $x = \overset{a}{7}$ to $x = \overset{b}{7+h}$

Possibilities:

- (a) 1
- (b) $2h$
- (c) h
- (d) $7 + 2h$
- (e) $14 + h$

$$\begin{aligned}
 \text{AROC} &= \frac{f(b) - f(a)}{b - a} \\
 &= \frac{f(7+h) - f(7)}{(7+h) - 7} \\
 &= \frac{((7+h) - 7)^2 + 5 - [(7-7)^2 + 5]}{h} \\
 &= \frac{(h^2 + 5) - 5}{h} \\
 &= \frac{h^2}{h} \\
 &= h
 \end{aligned}$$

8. Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 2x^2 + 6x + 3$ and $g(x) = 5x - 8$

Possibilities:

- (a) $(-\infty, \frac{8}{5}) \cup (\frac{8}{5}, \infty)$
- (b) $[\frac{8}{5}, \infty)$
- (c) $(-\infty, \frac{5}{8})$
- (d) $(-\infty, \infty)$
- (e) $\left[\frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{4}, \infty\right)$

$x \in \text{domain of } f$
 $x \in \text{domain of } g$
 $g(x) \neq 0$

Domain of $f \Rightarrow (-\infty, \infty)$
 Domain of $g \Rightarrow (-\infty, \infty)$
 $5x - 8 \neq 0$
 $5x \neq 8$
 $x \neq \frac{8}{5}$

9. Find $(f - g)(3)$ where $f(x) = 4x^2 - 9x - 7$ and $g(x) = 6x - 8$

Possibilities:

- (a) -8
- (b) 303
- (c) 4
- (d) 12
- (e) -24

$$\begin{aligned}(f - g)(3) &= f(3) - g(3) \\ &= [4(3)^2 - 9(3) - 7] - [6(3) - 8] \\ &= 4 \cdot 9 - 27 - 7 - 18 + 8 \\ &= 36 - 34 - 10 \\ &= 2 - 10 \\ &= -8\end{aligned}$$

10. A certain fungus grows in a circular shape. Its diameter in inches after t weeks is given below.

$$7 - 3e^{-8t} = \text{diameter}$$

Which of the following is an expression for the area covered as a function of time?

Possibilities:

- (a) $A(t) = \pi \left(\frac{7 - 3e^{-8t}}{2} \right)^2$
- (b) $t = \ln(3/7)/8$
- (c) $D(t) = 7 - 3e^{-8t}$
- (d) $A(t) = \pi 49 - 9e^{-64t}$
- (e) $A(t) = \pi t^2$

$$\begin{aligned}\text{radius} &= \frac{7 - 3e^{-8t}}{2} \\ \text{Area of Circle} &= \pi r^2 \\ A(t) &= \pi \left(\frac{7 - 3e^{-8t}}{2} \right)^2\end{aligned}$$

11. Suppose that the graph of $y = f(x)$ contains the point $(4, 9)$. Find a point that must be on the graph of $y = g(x)$ for $g(x) = 7 + 6f(x + 8)$.

Possibilities:

(a) $(11, 2)$

(b) $(-4, 61)$

(c) $(12, \frac{1}{3})$

(d) $(-\frac{2}{3}, 16)$

(e) $(-3, 47)$

$$4 = x + 8$$

$$-4 = x$$

$$g(-4) = 7 + 6f(-4 + 8)$$

$$g(-4) = 7 + 6 \cdot f(4)$$

$$= 7 + 6 \cdot 9$$

$$= 7 + 54$$

$$g(-4) = 61$$

$$(-4, 61)$$

$(4, 9)$
 $f(4) = 9$

12. Which sequence of transformations will transform the graph of the function f into the graph of the function g ?

$$f(x) = \sqrt{x} + 4 \quad g(x) = \sqrt{x-3} + 6$$

Possibilities:

(a) shift right by 3 then shift down by 2

(b) shift left by 3 then shift up by 2

(c) shift right by 3 then shift up by 2

(d) shift left by 3 then shift down by 2

(e) shift left by 2 then shift down by 3

$$f(x) = \sqrt{x} + 4$$

$$f(x-3) = \sqrt{x-3} + 4$$

$$f(x-3) + 2 = \sqrt{x-3} + 4 + 2$$

$$f(x-3) + 2 = \sqrt{x-3} + 6$$

$$f(x-3) + 2 = g(x)$$

↑ right 3 ↑ up 2

13. Use algebra to find the inverse of the given one-to-one function.

$$f(x) = (x^5 + 8)^7$$

Possibilities:

(a) $f^{-1}(x) = \sqrt[7]{\sqrt[5]{x} - 8}$

(b) $f^{-1}(x) = \sqrt[5]{\sqrt[7]{x} - 7}$

(c) $f^{-1}(x) = \sqrt[5]{\sqrt[7]{x} - 8}$

(d) $f^{-1}(x) = (x^7 + 8)^5$

(e) $f^{-1}(x) = x^{35} + 8$

$$y = (x^5 + 8)^7$$

$$x = (y^5 + 8)^7$$

$$\sqrt[7]{x} = y^5 + 8$$

$$\sqrt[7]{x} - 8 = y^5$$

$$\sqrt[5]{\sqrt[7]{x} - 8} = y = f^{-1}(x)$$

14. Use algebra to find the inverse of the given one-to-one function. $f(x) = \frac{4x}{9x + 7}$

Possibilities:

(a) $f^{-1}(x) = \frac{9x + 7}{4x}$

(b) $f^{-1}(x) = \frac{4x}{9x - 7}$

(c) $f^{-1}(x) = \frac{7x}{4x + 9}$

(d) $f^{-1}(x) = \frac{4}{9}x + 7$

(e) $f^{-1}(x) = \frac{7x}{4 - 9x}$

$$y = \frac{4x}{9x + 7}$$

$$x = \frac{4y}{9y + 7}$$

$$x(9y + 7) = 4y$$

$$9xy + 7x = 4y$$

$$9xy - 4y = -7x$$

$$y(9x - 4) = -7x$$

$$y = \frac{-7x}{9x - 4}$$

$$y = \frac{-7x}{-(4 - 9x)}$$

$$y = \frac{7x}{4 - 9x}$$

$$f^{-1}(x) = \frac{7x}{4 - 9x}$$

15. Write the given expression without using radicals.

$$\sqrt[7]{x^{11}}$$

Possibilities:

- (a) $x^{11/7}$
- (b) x^{-4}
- (c) $x^{7/11}$
- (d) x^4
- (e) $x^{11} - x^7$

$$(x^{11})^{1/7}$$

$$x^{11/7}$$

16. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week:	1	2	3	4	5	6
Frogs:	75	375	1875	9375	46875	234375

Which exponential growth model most closely matches the observations, if t is the week number?

Possibilities:

- (a) $5(75^{(t/7)})$
- (b) $5(75^t)$
- (c) $75(25^t)$
- (d) $15(5^t)$
- (e) $15(25^{(t/7)})$

$$P_0 = 15$$

$$P(t) = 15a^t$$

$$P(1) = 15a$$

$$75 = 15a$$

$$5 = a$$

$$P(t) = P_0 a^t$$

$$P(t) = 15(5^t)$$

17. Determine how much money will be in a savings account if the initial deposit was \$160 and the interest rate is 2.17% compounded continuously for 4 years, 11 months. (Round your answer to the nearest cent.)

Possibilities:

- (a) \$177.41
 (b) \$177.61
 (c) \$177.81
 (d) \$178.01
 (e) \$178.21

$$P(t) = P_0 e^{rt}$$

4 yrs, 11 months
 $= 4 \frac{11}{12}$ yrs.
 $t = 5 \frac{9}{12}$ yrs.

$$P\left(\frac{59}{12}\right) = 160 e^{(.0217)\left(\frac{59}{12}\right)}$$

$$= 178.01458\dots$$

$$\approx 178.01$$

$r = .0217$
 $P_0 = 160$

18. Translate the given logarithmic statement into an equivalent exponential one.

$$\log_4(9x + 7) = 16$$

$a \uparrow \quad \uparrow x \quad \uparrow y$

$$\log_a(x) = y \iff a^y = x$$

Possibilities:

- (a) $(16)^4 = 9x + 7$
 (b) $(9x + 7)^4 = 16$
 (c) $(4)^{16} = 9x + 7$
 (d) $(4)^{9x+7} = 16$
 (e) $(16)^{9x+7} = 4$

$$4^{16} = 9x + 7$$

19. Write the domain of the function $h(x) = \log(x - 12)$ in interval notation.

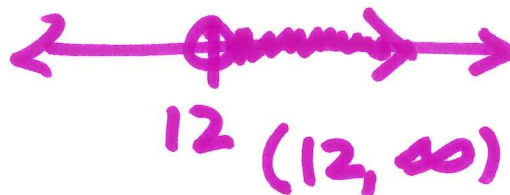
Possibilities:

- (a) $(-\infty, -12)$
- (b) $(12, \infty)$
- (c) $(-\infty, \infty)$
- (d) $(-\infty, 12]$
- (e) $(-\infty, 12) \cup (12, \infty)$

positive!

$$x - 12 > 0$$

$$x > 12$$



20. Write the given expression as a single logarithm.

$$4 \log(x) + \log(9y) - \log(7z)$$

Possibilities:

- (a) $\log(x^4 y^9 z^7)$
- (b) $\log\left(\frac{x^4(9y)}{7z}\right)$
- (c) $\log(4x(9+y) - 7 - z)$
- (d) $\log(4x + 9y - 7z)$
- (e) $\log\left(\frac{x^4 y^9}{z^7}\right)$

$$\log(x^4) + \log(9y) - \log(7z)$$

$$\log(x^4 \cdot 9y) - \log(7z)$$

$$\log\left(\frac{x^4 \cdot 9y}{7z}\right)$$

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt} \quad (\text{if compounded } n \text{ times per year})$$

$$P(t) = P_0 e^{rt} \quad (\text{if compounded continuously}).$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$