6 Using Technology Wisely

Concepts:

- Advantages and Disadvantages of Graphing Calculators

- How Do Calculators Sketch Graphs?

- When Do Calculators Produce Incorrect Graphs?

- The Greatest Integer Function

- Graphing Calculator Skills

    - Locating the Graph (TRACE AND TABLE)
    - Changing the Viewing Window (WINDOW)
    - Connected Mode vs. Dot Mode
    - The ZOOM Features
    - Finding Approximate Coordinates for Intersection Points
    - Finding Approximate Coordinates for x-intercepts

- The Intersection Method Revisited

- The Intercept Method Revisited

(Sections 2.1-2.2)

In this section we discuss the pros and cons of graphing calculators. So far, we have sketched the graphs of equations by plotting points and connecting the dots. This is precisely what graphing calculators do. One advantage of the calculator is that it can find and plot points very quickly. This means that in the time it takes you to plot 7 or 8 points, the calculator can plot hundreds. The more points that you plot, the more you know about the shape of the graph. Unfortunately, graphing calculators have one major drawback. Calculators do not know mathematics. There are subtleties to certain graphs that calculators always miss but good mathematics students always catch. Calculators are very helpful, but you always need to be smarter than your calculator. In this class, we will not be using TI-89 or TI-92 calculators.

## 6.1 Connecting the Dots. Is This Always a Good Plan?

As we mentioned earlier, graphing calculators usually work by plotting lots of points and connecting the dots. This technique is, in fact, how most students are first introduced to graphs of equations. This is not a bad first step if the graph is what you will call *continuous* in your Calculus class. Unfortunately, this approach can cause problems when there are breaks or *discontinuities* in the graph.

For the first example, you will need to recall the definition of the greatest integer function.

Definition 6.1
$[\![x]\!]$ is the **greatest integer** less than or equal to x.

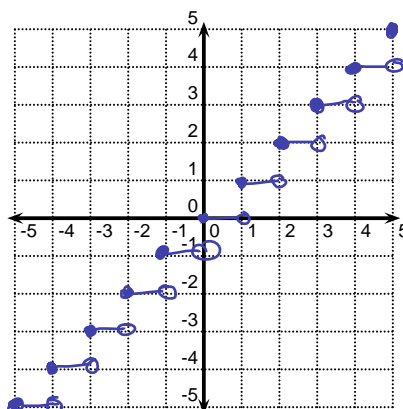For example $[\![2.5]\!] = 2$, $[\![-2.5]\!] = -3$, $[\![2]\!] = 2$, and $[\![-2]\!] = -2$.
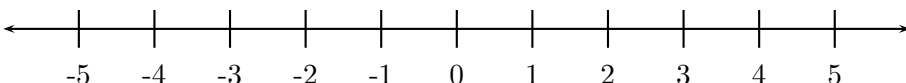
Example 6.2 (Plotting Points Can Lead to Errors)
Sketch the graph of $y = [\![x]\!]$.

Some solution points:

x	y	Solution Point
2.5	2	(2.5, 2)
2.9	2	(2.9, 2)
2.999	2	(2.999, 2)
3	3	(3, 3)
-2.5	-3	(-2.5, -3)

Should we connect the dots to obtain the graph of $y = [\![x]\!]$? Explain.

We should connect those dots on the intervals between integers, but not the vertical 'steps'. If we did, we wouldn't have the graph of a function.

The previous example shows us that the calculator would produce an incorrect graph if it connect the solution points. It is possible to tell the calculator that it should not connect the dots. This requires that you are **SMARTER THAN YOUR CALCULATOR.** As we will see later, even this does not solve all of our problems. It is important to remember that **graphing calculators produce approximate graphs at best**.

## 6.2 Basic Instructions for the Graphing Calculator

In order to see an approximate graph of an equation in the variables x and y, you need to:

1. Solve the equation for y in terms of x.

2. Hit $\boxed{y=}$ on your calculator.

3. Enter the equation you wish to graph. You should use the $\boxed{X, T, \theta, n}$ key for X.

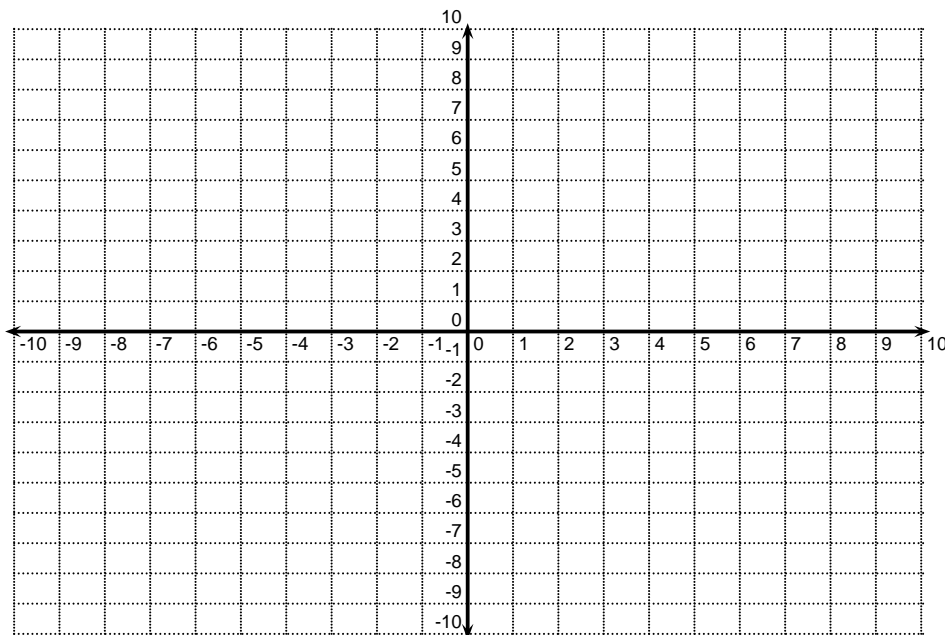4. Hit the $\boxed{\text{GRAPH}}$ key.

6.3 Graphing Calculator Examples

Example 6.3 (A Basic Example)
Sketch the graph of $y = x^5 + 3x^2 + 1$.
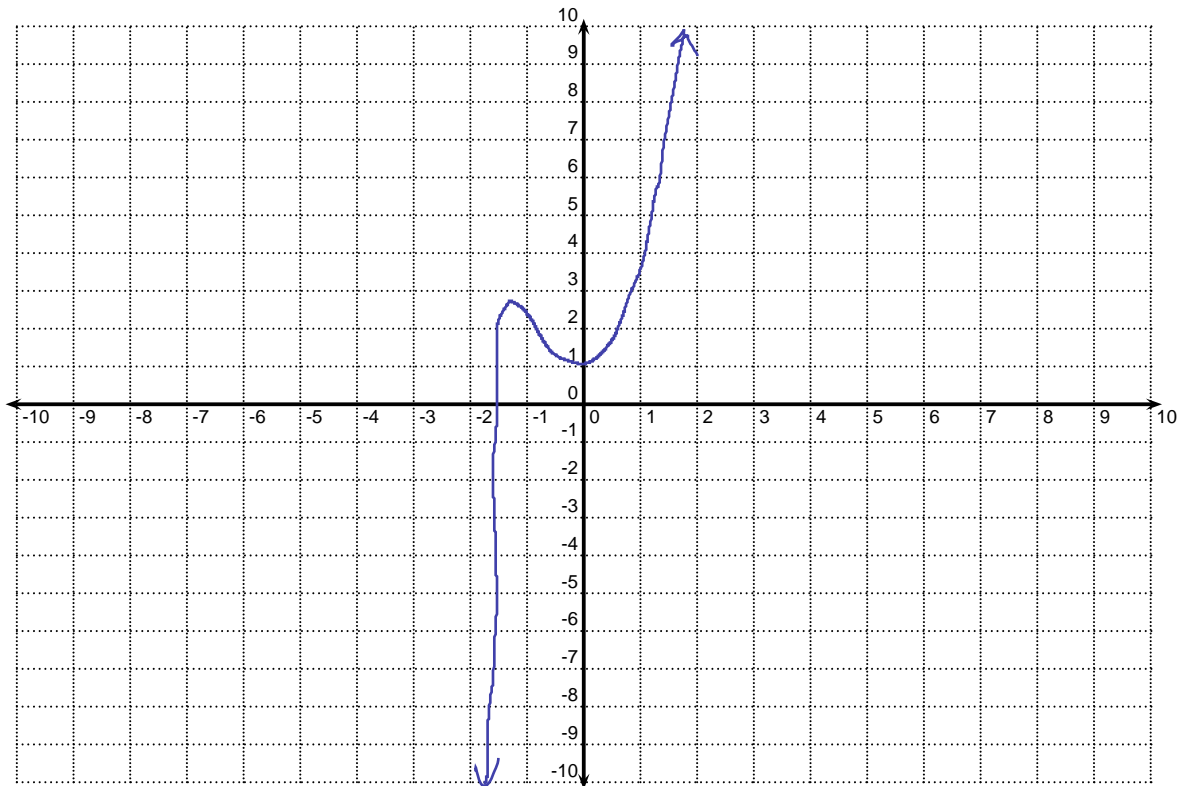
Example 6.4 (Changing the Viewing Window)
Sketch the graph of $y = 100\sqrt[5]{x}$

Why is this not a great picture? *There's nothing there to graph.*

How do you find the picture? It can help to know what points are being plotted. This can be accomplished with the 2nd TABLE key.

Seeing the Coordinates of Points on a Graph

There are two options for seeing points on the graph.

1. Looking at values in a table.

   (a) Hit 2nd TABLE. (TABLE is above the GRAPH key.)

   (b) Use the UP and DOWN arrows to see some of the solution points.

   (c) The default for TABLE is to start at $x = 0$ and to change or increment the x value by 1 unit at each step. This can be modified by hitting 2nd TBLSET. (TBLSET is above the WINDOW key.) Then you can modify TblStart and ΔTbl to change the initial x value and the increment value, respectively.

2. Looking at points along the graph.

   (a) Hit TRACE

   (b) Use the right and left arrow keys to move along a graph.

   (c) Use the up and down arrow keys to move from one graph to a different graph.

Once you have seen some of the solution points, you will have a better idea where you need to look for the graph. The standard viewing window for graphs on most calculators shows you x values between -10 (Xmin) and 10 (Xmax) and y values between -10 (Ymin) and 10 (Ymax). The graphs also contain a tick mark every 1 unit (Xscl,Yscl) on the x and y axes. These values can be easily changed to improve the quality of the picture.

Changing the Viewing Window.

1. Hit the $\boxed{\text{WINDOW}}$ key.

2. Change the following values as necessary.

    (a) Xmin
    (b) Xmax
    (c) Xscl
    (d) Ymin
    (e) Ymax
    (f) Yscl

3. To exit, hit $\boxed{\text{2nd}}$ $\boxed{\text{QUIT}}$. (QUIT is above the $\boxed{\text{MODE}}$ key.)

A better graph:
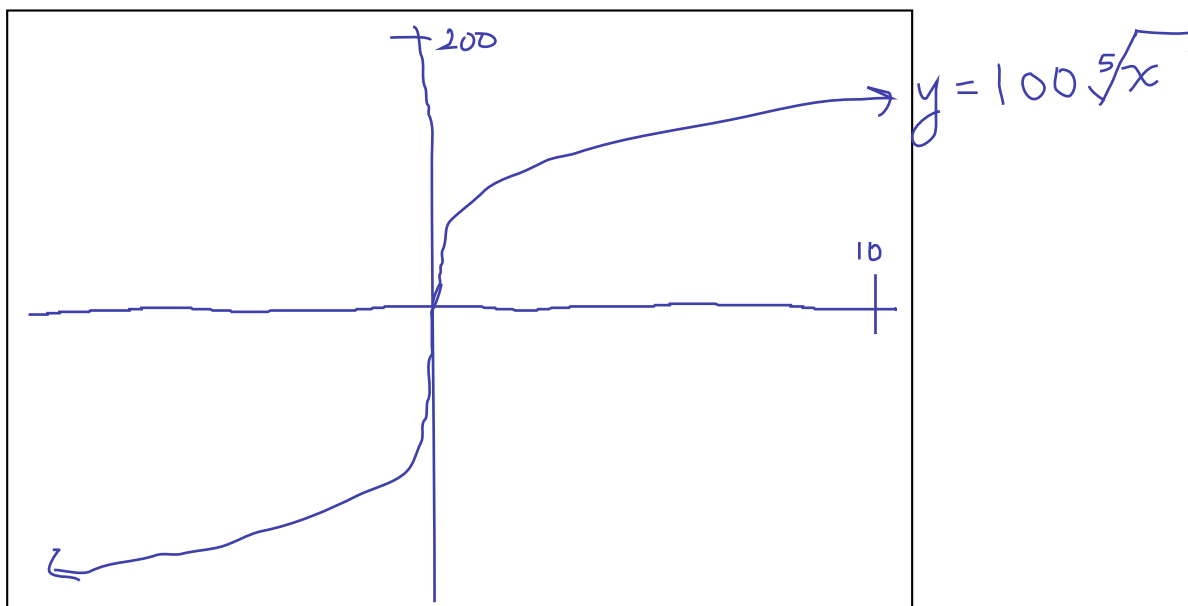
When you sketch a graph in this class, **ALWAYS LABEL THE VIEWING WINDOW.**

Example 6.5 (Graphing Circles)

Sketch the graph of $(x + 1)^2 + y^2 = 4$.

Should you be using technology to sketch this graph? Explain.

This is a circle with center $(-1, 0)$ and radius 2. Whether or not to use technology depends on what you need. You already know a lot of information.

What do you need to enter into the calculator to sketch this graph?

$$y^2 = 4 - (x + 1)^2$$

$$y = \pm\sqrt{4 - (x+1)^2}$$

$$Y_1 = \sqrt{4 - (x+1)^2}$$

$$Y_2 = -\sqrt{4 - (x+1)^2}$$

Let's see what goes wrong when we use technology to sketch this graph.

We will first look at this graph in a standard viewing window ($x \in [-10, 10]$, $y \in [-10, 10]$). We know that we can return to a standard viewing window by changing the viewing window. There is another alternative that is faster.

> **The Standard Viewing Window:**
> To return to the standard viewing window ($x \in [-10, 10]$, $y \in [-10, 10]$), you can:
>
> - follow the procedure to change the viewing window OR
>
> - Hit ZOOM. Use the arrow keys to select ZStandard. Hit ENTER.
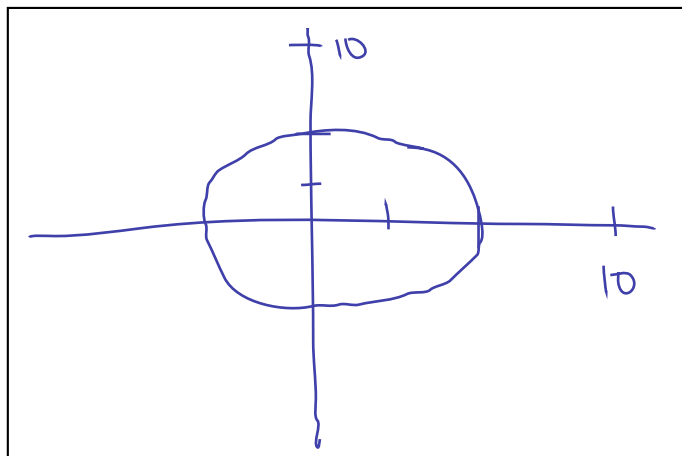
The problem with this picture is that the screen is shorter than it is wide. The distance between tick marks on the x-axis is greater than the distance between tick marks on the

y-axis. We can make these distances equal by hitting $\boxed{\text{ZOOM}}$ and using the arrow keys to select ZSquare.

Sketch the graph after you hit $\boxed{\text{ZOOM}}$→ZSquare. Use the Window key to find the new viewing window of the graph. What is the approximate ratio of the width of the screen to the height of the screen?
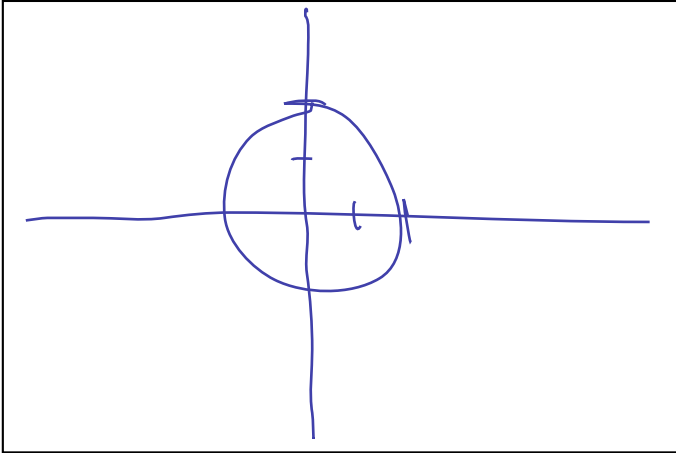
The $\boxed{\textbf{ZOOM}}$ Key:
The $\boxed{\text{ZOOM}}$ key has many useful features that we have not discussed. One, in particular is the ZBox feature. This feature allows you to place a rectangle around a portion of the graph. The calculator will then zoom in on the portion of the graph contained in the rectangle.
To use the ZBox feature:

1. Hit $\boxed{\text{ZOOM}}$.

2. Select ZBox.

3. Use the arrow keys to position the cursor at one corner of the rectangle.

4. Hit $\boxed{\text{ENTER}}$.

5. Use the arrow keys to position the cursor at the opposite corner of the rectangle.
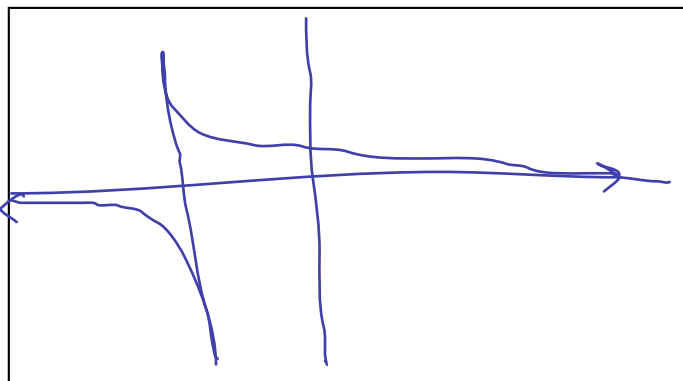
6. Hit $\boxed{\text{ENTER}}$.

The $\boxed{\text{ZOOM}}$ key has several other features including Zoom In and Zoom Out. You should experiment with these to see how they work.

Example 6.6

Sketch the graph of $y = \dfrac{1}{x+4}$.
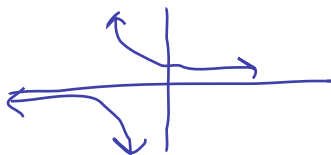
Let's see what goes wrong when we use technology to sketch this graph.
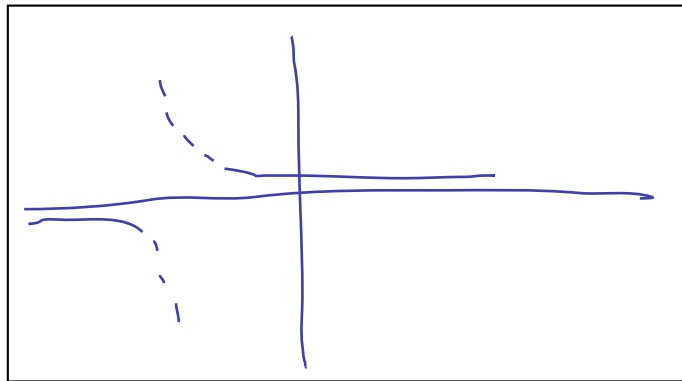
The vertical line should not be there.

Connected vs. Dot Mode:

In Connected mode, the calculator will connect the points that it plots. In Dot mode, the calculator will not connect the points.

To change between Connected mode and Dot mode:

1. Hit $\boxed{\text{MODE}}$,

2. Use the arrow keys to highlight *Connected* or *Dot*.

3. Hit $\boxed{\text{ENTER}}$

Change to Dot mode and sketch the new graph.

Is this a better graph? Is this graph correct? Explain.

The vertical line does not show up but there are large parts missing.

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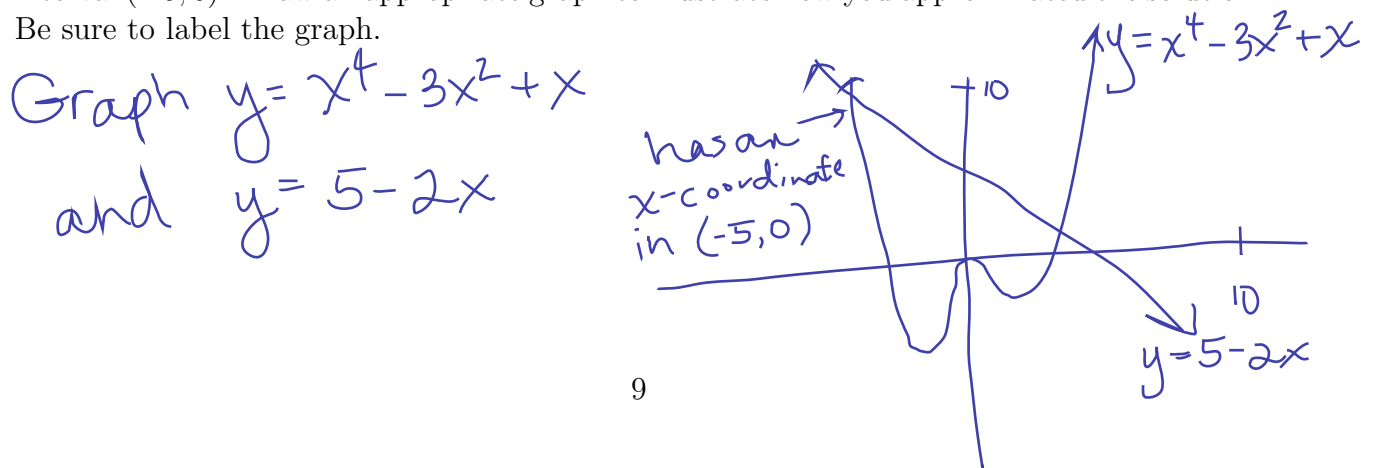## 6.4 Using Technology to Find Approximate Solutions of Equations in One Variable

Recall that the Intercept Method and the Intersection Method are two graphical methods for approximating the solutions of an equation in one variable. These methods should not be used to solve equations when we have a simple algebraic technique for finding the exact solutions of and equation. There are two functions on the graphing calculator that can be used to approximate the coordinates of intersection points and x-intercepts.

Finding Approximate Coordinates for Intersection Points:

1. Enter the equation for the first graph in Y_1.

2. Enter the equation for the second graph in Y_2.

3. Hit $\boxed{\text{GRAPH}}$. Make sure that you can see the intersection point on the graph. If you cannot see it, adjust the viewing window.

4. Hit $\boxed{\text{2nd}}$ CALC. (CALC is above the $\boxed{\text{TRACE}}$ key.)

5. Use the arrow keys to select *intersect*. Hit $\boxed{\text{ENTER}}$.

6. If necessary, use the up and down arrow keys to select the first graph. Hit $\boxed{\text{ENTER}}$. (You may not need to use the arrow keys. The calculator will probably place the cursor on the first graph.)

7. If necessary, use the up and down arrow keys to select the second graph. Hit $\boxed{\text{ENTER}}$. (You may not need to use the arrow keys. The calculator will probably place the cursor on the second graph.)

8. Use the right and left arrow keys to place the cursor close to the intersection point. (This is your Guess.) Hit $\boxed{\text{ENTER}}$.

Example 6.7 (The Intesection Method)
Use the Intersection Method to approximate the solution of $x^4 - 3x^2 + x = 5 - 2x$ in the interval $(-5, 0)$. Draw an appropriate graph to illustrate how you approximated the solution. Be sure to label the graph.

Graph $y = x^4 - 3x^2 + x$

and $y = 5 - 2x$

hasan →
x-coordinate
in $(-5, 0)$

$y = x^4 - 3x^2 + x$

$y = 5 - 2x$

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$x \approx -2.293241$

Example 6.8 (The Intecept Method)

Use the Intercept Method to approximate the solution of $x^4 - 3x^2 + x = 5 - 2x$ in the interval $(0, 5)$. Draw an appropriate graph to illustrate how you approximated the solution. Be sure to label the graph.

Graph

$$y = x^4 - 3x^2 + 3x - 5$$

$x \approx 1.7171574$

has x-coordinate in $(0,5)$
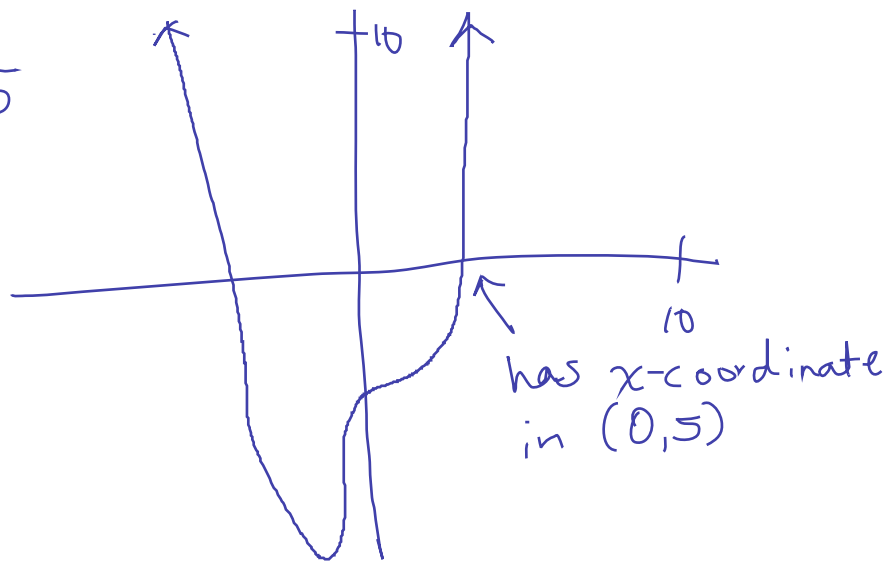
Example 6.9 (Choosing a Method for Solving an Equation)

Which of the following equations should be solved algebraically and which should be solved graphically?

$$3 - x + 2x^2 = 5 + x \qquad \text{algebraically}$$

$$x^5 + 3\sqrt{x} = 7 \qquad \text{graphically}$$

$$\frac{1}{x+2} = 5x \qquad \text{alg.}$$

$$x^4 + 2x^2 - 1 = 0 \qquad \text{alg.}$$

$$x^4 + 2x - 1 = 0 \qquad \text{graphically}$$