Directions:

- This is a two hour exam. Clearly print your name on the first page and the top of the third page (second piece of paper). No books, notes, internet connection, or cell phone can be used during this exam. Any scratch paper must be provided to you by the proctor and turned in with the exam. A calculator maybe used; however, the calculator cannot have a Computer Algebra System (CAS) or a QWERTY keyboard. When you have completed the exam:
  1) Turn in the entire exam (including cover page, and any scratch papers) to the proctor
  2) Show your ID to the proctor
  3) Sign the "Sign Out Sheet"
- All answers must be fully filled in on the front page, like so:

  A B C D E

- The exam is out of 100 total points; however, it is possible to earn up to 115 points (5 points for each of the 23 questions). Only this front page will be graded and no partial credit will be awarded. Consequently, please double check to make sure that you have marked the answer you desire. Good Luck!

#1 ● B C D E
#2 A B C D ●
#3 A ● C D E
#4 A B ● D E
#5 ● B C D E
#6 A B C D ●
#7 A B ● D E
#8 A B ● D E
#9 A ● C D E
#10 ● B C D E
#11 A B ● D E
#12 A B ● D E
#13 ● B C D E
#14 A ● C D E
#15 A B C ● E
#16 A B ● D E
#17 A ● C D E
#18 A ● C D E
#19 ● B C D E
#20 ● B C D E
#21 A B C D ●
#22 A B ● D E
#23 A ● C D E

Name (Print):______________________________

Section Number:______________________________

Section	Instructor	Class Start Time	Exam Location
001	Drew Butcher	MWF 8:00 AM	BS 116
002	Drew Butcher	MWF 10:00 AM	BS 107
003	Drew Butcher	MWF 1:00 PM	CB 118
004	Robert Wolf	MWF 9:00 AM	CB 122
005	Robert Wolf	MWF 11:00 AM	CB 122
006	Ian Barnett	TR 11:00 AM	CB 114
007	Ian Barnett	TR 12:30 PM	CB 114
008	Devin Willmott	TR 2:00 PM	CB 110
009	Devin Willmott	TR 3:30 PM	CB 110

UK: "Go CATS" Name:________________________________ Section:________

1. (5 points) A bakery has a weekly fixed cost of \$1000 and it costs the bakery \$2 to make a pie. Express C, the total weekly cost of operating the bakery as a function of p, the number of pies made in a week.

   A. $C = 1000 + 2p$
   B. $C = 1000 - 2p$
   C. $C = 2p - 1000$
   D. $C = 1000p + 2p$
   E. $C = 1000p + 2$

   Cost = fixed cost + variable cost
   C = 1000 + 2p

2. (5 points) Let $f(x)$ be a function. Which of the following is always true?

   A. There is exactly one input for each output of $f(x)$
   B. $f(x)$ can take any number as an input
   C. If $f(a) = b$, then $f(b) = a$
   D. $f(a + b) = f(a) + f(b)$
   E. The graph of $f(x)$ intersects any vertical line at most once

3. (5 points) What is the domain of $f(x)$, where $f(x) = \dfrac{x^2 + 7}{x - 2}$?

   A. $(-\infty, \infty)$
   B. $(-\infty, 2) \cup (2, \infty)$
   C. $[0, \infty)$
   D. $[-7, \infty)$
   E. $(-\infty, -2) \cup (-2, \infty)$

   can not have $x - 2 = 0$
   $x = 2$

   Domain is all real # except $x = 2$

   2
   $(-\infty, 2) \cup (2, \infty)$

4. (5 points) The following table describes all the inputs and outputs of a function f.

Input	-2	-1	0	2
Output	17	4	4	-1

← Domain element
← Range element

Which of the following accurately describes f?

   A. 4 is in the ~~domain~~ range of f
   B. 2 is ~~not~~ in the domain of f
   C. 17 is in the range of f :)
   D. -1 is ~~not~~ in the range of f
   E. f does not have a domain or a range, because f is not a function

   f is a function each input corresponds to one output

5. (5 points) What is the domain of $g(x)$, where $g(x) = \dfrac{6}{\sqrt{x^2+1}}$?

   A. $(-\infty, \infty)$
   B. $[1, \infty)$
   C. $(1, \infty)$
   D. $(-1, \infty)$
   E. $[-1, \infty)$

1st Can not have $\sqrt{x^2+1} = 0$

$x^2+1 = 0^2$

$x^2+1=0$

$x^2=-1$

Impossible!!

This says the fraction part produces no restrictions

2nd Observe $x^2 \geq 0$

so $x^2+1 \geq 0+1$

$x^2+1 \geq 1 > 0$

So the radical part produces no restriction

Consequently domain is all reals.

6. (5 points) Suppose you have a square tank with side length of 2 feet. The tank is filled such that the height of the water rises at a rate of 3 ft per second. Assuming that the tank starts out empty, what is the volume of water in the tank as a function of time?

   A. $V(t) = \dfrac{t}{4}$
   B. $V(t) = \dfrac{t}{3}$
   C. $V(t) = 3t$
   D. $V(t) = 4t$
   E. $V(t) = 12t$

Let t be the length of time (seconds) water has poured into the tank

$V = lwh$

Note $l = 2$
$w = 2$
$h = 3t$

$V(t) = 2 \cdot 2 \cdot 3t$
$V(t) = 12t$

7. (5 points) The tortoise and the hare compete in a 1 mile race. The tortoise runs 5 mph and the hare runs 20 mph. How long can the hare nap before losing the race?

   A. 1 minute
   B. 5 minutes
   C. 9 minutes
   D. 15 minutes
   E. 20 minutes

Let x be the hare's nap time

Recall: $d = vt$

Time for tortoise to complete the race

$d = vt$

$\frac{1}{5} = \frac{5t}{5}$

$t = \frac{1}{5}$ hours

$t = 0.20$ hours

Time for hare to complete the race without a nap.

$d = vt$

$\frac{1}{20} = \frac{20t}{20}$

$t = \frac{1}{20}$ hours

$t = 0.05$ hour

Note

hare's nap time + hare's race time $\leq$ tortoise's race time

$x + 0.5 \leq 0.20$

$x \leq 0.15$ hour $= 9$ minute

8. (5 points) Find the average rate of change of $f(x) = x^2 + 3x + 1$ between $x = 1$ and $x = 2$.

   A. -2
   B. -6
   C. 6
   D. 2
   E. 0

a b

$$\frac{f(b)-f(a)}{b-a} = \frac{f(2)-f(1)}{2-1} = \frac{11-5}{2-1} = \frac{6}{1} = 6$$

Note

$f(2) = 2^2 + 3(2) + 1$
$= 4 + 6 + 1$
$= 11$

$f(1) = 1^2 + 3(1) + 1$
$= 1 + 3 + 1$
$= 5$

9. (5 points) Find the domain of $\left(\frac{g}{f}\right)(x)$, where

polynomials

$$f(x) = x - 3 \qquad \text{and} \qquad g(x) = x + 3.$$

A. $(-\infty, \infty)$
B. $(-\infty, 3) \cup (3, \infty)$
C. $(-\infty, -3) \cup (-3, \infty)$
D. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
E. None of the above

Domain of f is all reals
Domain of g is all reals
We can not have $f(x) = 0$
$x - 3 = 0$
$x = 3$

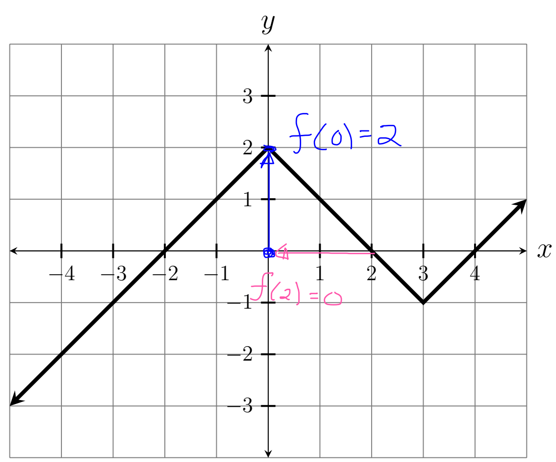
3

10. (5 points) Let $f(x) = |x - 3|$ and $g(x) = 1 - x^2$. Determine $(f \circ g)(2)$.

A. 6
B. 0
C. -3
D. -2
E. None of the above

$(f \circ g)(2) = f(g(2)) = f(1 - 2^2) = f(1-4) = f(-3)$
$= |-3-3| = |-6| = 6$

11. (5 points) The graph of f is shown below. Using this graph compute $f(f(0))$.



A. -4
B. -2
C. 0
D. 2
E. None of the above

$f(f(0)) = f(2) = 0$

12. (5 points) Give the graph transformations that change $f(x) = x^3 + x$ into $g(x) = 3(x-2)^3 + 3x - 6$.

    A. Translate right by 2 then scale vertically by a factor of $\frac{1}{3}$.

    B. Scale vertically by a factor of 3 then translate left by 2.

    C. Scale vertically by a factor of 3 then translate right by 2.

    D. Translate right by 2 then scale horizontally by a factor of 3.

    E. Translate right by 2.

Vertically scale by 3
$3f(x) = 3x^3 + 3x$
Shift Right by 2
$3f(x-2) = 3(x-2)^3 + 3(x-2)$
$= \underbrace{3(x-2)^3 + 3x - 6}_{g(x)}$

13. (5 points) The monthly cost of running a local coffee shop can be approximated by a linear cost function $C(x) = Ax + F$ where A is the average cost of making a drink, x is the number of drinks sold, and F is the monthly fixed cost of the business. If the rent goes up by \$1000 a month then how does the graph of $C(x)$ transform?

    A. Translates up by 1000

    B. Translates down by 1000

    C. Translates left by 1000

    D. Translates right by 1000

    E. Scales vertically by a factor of 1000

Add 1000 to $C(x)$ → shifts graph up 1000

14. (5 points) $f(x) = x^2$ is transformed into $g(x) = (x+3)^2 + 1$. Find where the point $(0,0)$ on the graph of f moves to under the transformation.

    A. $(3, 1)$

    B. $(-3, 1)$

    C. $(3, -1)$

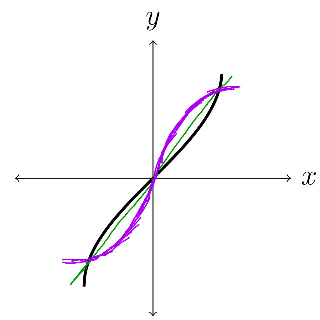
    D. $(1, -3)$

    E. $(-1, 3)$

$f(x) = x^2 \xrightarrow{\text{up } 1} f(x) + 1 = x^2 + 1 \xrightarrow{\text{left } 3} f(x+3) + 1 = \overbrace{(x+3)^2 + 1}^{g(x)}$

$(0,0) \xrightarrow[\text{add 1 to y}]{} (0, 1) \xrightarrow[\text{Subtract 3 from x}]{} (-3, 1)$

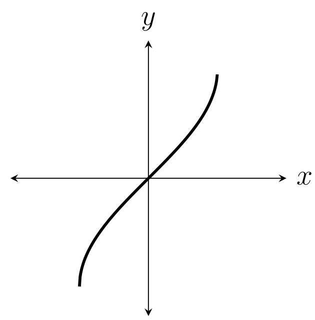
15. (5 points) Consider the following graph:



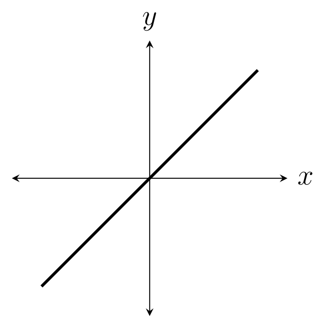
Reflect over $y = x$ to obtain inverse

Which of the following is the inverse to the given graph?

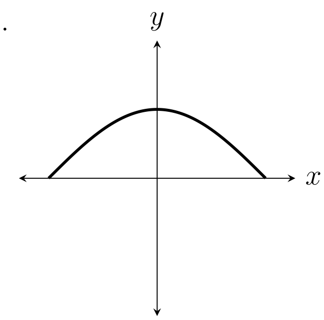
A.



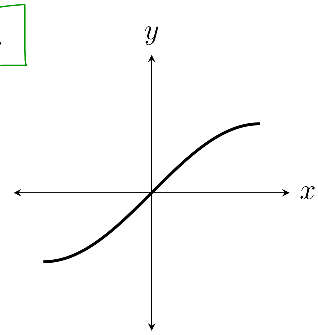
B.



C.

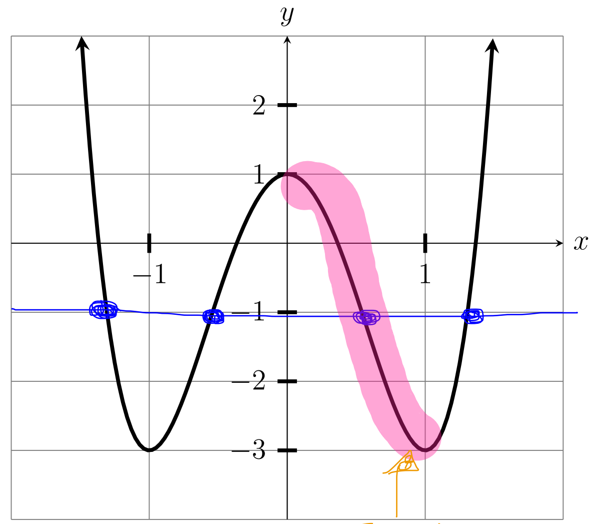


$\boxed{\text{D.}}$



E. The given graph is not invertible

16. (5 points) Consider the graph of $f(x)$:



Does not pass horizontal line test

Just this piece passes the horizontal line test

On which of the following intervals does f have an inverse?

A. $(-\infty, \infty)$

B. $(-1, 1)$

C. $(0, 1)$

D. $(0, \infty)$

E. $(-\infty, 1)$

17. (5 points) Use the given table to find the value of $f^{-1}(f(0) + 1)$.

x	$f(x)$
-1	2
0	2
1	3
2	2
3	1

First observe $f(0) = 2$

So $f^{-1}(f(0)+1) = f^{-1}(2+1)$
$= f^{-1}(3)$

Since $f(1) = 3$ we know $f^{-1}(3) = 1$

A. 0

B. 1

C. 2

D. 3

E. There is not enough information to determine the answer.

18. (5 points) Write the radical expression $\sqrt[3]{x^7}$ without any radicals.

A. $x^{\frac{3}{7}}$

B. $x^{\frac{7}{3}}$

C. x^{21}

D. x^4

E. None of the above

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$x^{\frac{7}{3}} = \sqrt[3]{x^7}$

19. (5 points) Compute the average rate of change of the function $f(x) = 5^x$ from $x = \overset{a}{1}$ to $x = \overset{b}{6}$.

A. 3, 124

B. 15, 624

C. $\dfrac{1}{3,124}$

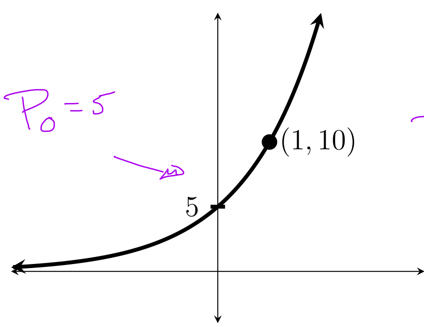
D. $\dfrac{1}{15,624}$

E. None of the above

$\dfrac{f(b)-f(a)}{b-a} = \dfrac{f(6)-f(1)}{6-1} = \dfrac{5^6-5^1}{6-1} = \dfrac{15625-5}{5} = \dfrac{15620}{5} = 3,124$

20. (5 points) The graph below represents an exponential growth function $P(t) = P_0 a^t$. Determine the rule of the function.

$P_0 = 5$



$P(1) = 10$

A. $P(t) = 5(2^t)$

B. $P(t) = 2(5^t)$

C. $P(t) = 10(5^t)$

D. $P(t) = 5(10^t)$

E. None of the above

$P(t) = 5a^t$

$P(1) = 10$

and $P(1) = 5a^1$

we have $\dfrac{10}{5} = \dfrac{5a^1}{5}$

$a = 2$

So $P(t) = 5 \cdot 2^t$

21. (5 points) Evaluate $\log_7(343)$.

A. 2

B. $\ln(343)$

C. $\log(343)$

D. 49

E. 3

Note $343 = 7^3$

so $\log_7(343) = \log_7(7^3) = 3$

22. (5 points) Simplify $\ln(x) + \ln(x^2) - 3\ln(x)$.

A. $\ln(x)$

B. 1

C. 0

D. $\ln(x^2/3)$

E. None of the above

$= \ln(x) + \ln(x^2) - \ln(x^3)$

$= \ln(x \cdot x^2) - \ln(x^3)$

$= \ln(x^3) - \ln(x^3)$

$= \ln\left(\frac{x^3}{x^3}\right)$

$= \ln(1) = 0$

23. (5 points) Define $P(t) = e^t$. What is the value of $P^{-1}(45)$?

A. $\log(45)$

B. $\ln(45)$

C. 45

D. $\log_{45}(e)$

E. There is no such value

$P^{-1}(t) = \ln(t)$

so $P^{-1}(45) = \ln(45)$

Formula Sheet

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1+\frac{r}{n}\right)^{nt} \qquad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0 e^{rt} \qquad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a,b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$