Name: _____ **Section:** _____

MA 109 Spring 2014

Exam 4 May 5, 2014

Directions:

- Do not remove this page—you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.
- The exam consists of multiple choice and short answer questions. Record your answers on this page by filling in the appropriate selection, for example:

  (A) B C D E.
- The exam is out of 100 total points: 5 points for each of 25 multiple choice questions. There is a possibility for up to 125 points on the exam (but 25 of these count as extra credit). **Only** this front page will be graded and **no partial credit** will be awarded. It is recommended that you check your work!

1. (D)
2. (B)
3. (B)
4. (C)
5. (E)
6. (C)
7. (D)
8. (A)
9. (D)
10. (C)
11. (E)
12. (D)
13. (C)
14. (C)
15. (B)
16. (E)
17. (A)
18. (E)
19. (C)
20. (C)

For grading use:

Number Correct (out of 25 questions)	**Total Points Earned** (questions worth 5 points each)

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

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Multiple Choice: Show your work in the space below and shade the correct answer on the front page for each of the following.

1. Let

$$f(x) = \begin{cases} x+1 & \text{if } x \le -3 \\ x^2 - 3 & \text{if } -3 < x \le 2 \\ -2x + 5 & \text{if } x > 2 \end{cases}$$

Find $f(-4)$.

Choices:

(a) 6
(b) -2
(c) -3
(d) 13
(e) 0

$f(-4) = -4 + 1 = -3$

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2. Solve for z.

$$-2z^2 - 6z + 1 = 0$$

$2z^2 + 6z - 1 = 0$

Choices:

(a) $\frac{-6}{4} \pm \sqrt{11}$
(b) $\frac{-3 \pm \sqrt{11}}{2}$
(c) $\frac{-6 \pm \sqrt{28}}{4}$
(d) $\frac{6 \pm \sqrt{28}}{4}$
(e) $\frac{-2 \pm \sqrt{36}}{6}$

$\frac{-6 \pm \sqrt{6^2 - 4(2)(-1)}}{2(2)} = \frac{-6 \pm \sqrt{36+8}}{4} = \frac{-6 \pm \sqrt{44}}{4}$

$= \frac{-6 \pm 2\sqrt{11}}{4} = \frac{2(-3 \pm \sqrt{11})}{4 \cdot 2} = \frac{-3 \pm \sqrt{11}}{2}$

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3. Solve for r.

$$(3r - 18)(r^2 - 9) = 0$$

Choices:

(a) The only real solutions are 6 and 3.

(b) The only real solutions are 6 and ± 3.

(c) The only real solutions are 3 and 9.

(d) The are no real solutions.

(e) The only real solutions are ± 3.

$3r - 18 = 0$ $\quad r^2 - 9 = 0$

$3r = 18$ $\quad r^2 = 9$

$r = \frac{18}{3}$ $\quad r = \pm 3$

$r = 6$

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4. For which of the following equations is the number 2 a solution?

Choices:

(a) $4(3 - x) = 12$

(b) $\frac{4}{x} + 2 = \frac{1}{x - 3}$

(c) $2x^2 - 8 = 0$

(d) $2x^2 + 2x - 8 = 16$

(e) $|2x| = -6$

$4(3-2) = 4 \cdot 1 = 4 \neq 12$

$\frac{4}{2} + 2 = 2 + 2 = 4 \stackrel{?}{=} \frac{1}{2-3} = \frac{1}{-1} = -1$ X

$2(2)^2 - 8 = 2 \cdot 4 - 8 = 8 - 8 = 0$

$2(2)^2 + 2(2) - 8 = 2 \cdot 4 + 4 - 8 = 8 + 4 - 8 = 4 \neq 16$

Impossible

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5. Use the Intersect or Intercept Method to approximate all real solutions to the equation below using your calculator.

$$x^5 - x^2 + 3x = 3 + x^2$$

Choices:

(a) $x \approx 1.822$

(b) $x \approx -2.112$

(c) $x \approx -0.632$

(d) $x \approx 2.260$

(e) $x \approx 1.175$

6. Let $f(x) = 3^x$. Which of the following is $f^{-1}(27)$?

Choices:

(a) 27
(b) $\frac{1}{27}$
(c) 3
(d) $\frac{1}{3}$
(e) -1

$f^{-1}(y) = x$ means $f(x) = y$
$f^{-1}(27) = x$
$f(x) = 27$
$3^x = 27$
$3^x = 3^3$
$x = 3$

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7. Solve the inequality $|x - 2| > 5$.

Choices:

(a) $(-3, 7)$
(b) $(-5, 5)$
(c) $(-\infty, -2) \cup (5, \infty)$
(d) $(-\infty, -3) \cup (7, \infty)$
(e) $(-\infty, -3] \cup [7, \infty)$

$w = x - 2$
$|w| > 5$
$w > 5$ and $w < -5$
$x - 2 > 5$ and $x - 2 < -5$
$x > 7$ $\quad x > -3$

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8. Find an equation for the line through the points $(-4, 1)$ and $(5, 10)$.

Choices:

(a) $y - 1 = (x + 4)$
(b) $y - 4 = -9(x - 1)$
(c) $y + 5 = -9(x - 10)$
(d) $y + 4 = \frac{1}{9}(x - 5)$
(e) $y - 5 = -\frac{9}{5}(x - 5)$

$m = \frac{10 - 1}{5 - (-4)} = \frac{9}{9} = 1$
$y - 1 = 1(x + 4)$

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9. Solve the following system of equations. $\begin{cases} 4x + 2y = 7 \\ 2x + 2y = 8 \end{cases}$ $\rightarrow$ $4x + 2y = 7$, $-2x - 2y = -8$, $2x = -1$, $x = -\frac{1}{2}$

Choices:

(a) The system has no solution.

(b) The system has infinitely many solutions one of which is $(\frac{1}{2}, \frac{9}{2})$

(c) Every point is a solution to the system.

(d) The only solution is $(-\frac{1}{2}, \frac{9}{2})$.

(e) The only solution is $(-\frac{1}{2}, 3)$.

$2x+2y=8 \rightarrow x+y=4$, $y = 4-x$, $y = 4-(-\frac{1}{2}) = \frac{8}{2}+\frac{1}{2} = \frac{9}{2}$

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10. The number of bacteria in a culture is modeled by the function $n(t) = 100e^{0.5t}$ where t is measured in hours. When will the number of bacteria reach 2500? Round your answer to the nearest tenth of an hour.

Choices:

(a) About 13.2 hours

(b) About 5.9 hours

(c) About 6.4 hours

(d) About 2.8 hours

(e) About 3.7 hours

$\frac{2500}{100} = \frac{100e^{0.5t}}{100}$, $25 = e^{0.5t}$, $\ln(25) = \ln(e^{0.5t})$, $\ln(25) = 0.5t$, $t = \frac{\ln(25)}{0.5} = 6.437$

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11. Let $f(x) = 3x^2 - x - 1$. Find $\frac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Choices:

(a) 1

(b) $3h$

(c) $\frac{6xh + 3h^2 - 1}{h}$

(d) $18x + 9h$

(e) $6x + 3h - 1$

$\frac{[3(x+h)^2-(x+h)-1]-[3x^2-x-1]}{h} = \frac{3(x^2+2xh+h^2)-x-h-1-3x^2+x+1}{h}$

$= \frac{3x^2+6xh+3h^2-x-h-1-3x^2+x+1}{h} = \frac{6xh+3h^2-h}{h}$

$= \frac{h[6x+3h-1]}{h} = 6x+3h-1$

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12. Find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x) = 3x^3 - 2x^2 - x + 1$ is divided by $x - 2$.

Choices:

(a) $Q(x) = 3x^2 - 8x + 15, \quad R(x) = -29$

(b) $Q(x) = \frac{1}{3}x^2 + 4x + 7, \quad R(x) = 0$

(c) $Q(x) = 3x^2 + 4x + 9, \quad R(x) = -17$

(d) $Q(x) = 3x^2 + 4x + 7, \quad R(x) = 15$

(e) $Q(x) = 3x^2 + 4x, \quad R(x) = 7x + 1$

$$\begin{array}{r|l} x-2 & 3x^2+4x+7 \\ \hline & 3x^3-2x^2-x+1 \\ & -3x^3+6x^2 \\ \hline & 4x^2-x+1 \\ & -4x^2+8x \\ \hline & 7x+1 \\ & -7x+14 \\ \hline & 15 \end{array}$$

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13. Determine the end behavior of the graph of $y = -x^5 + 2x - 6$.

Choices:

(a) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

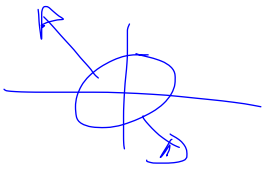
(b) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(c) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(d) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(e) None of the above.

negative odd



$y \to -\infty$ as $x \to \infty$

$y \to \infty$ as $x \to -\infty$

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14. What is the average rate of change of $f(x) = -5x - 3$ with respect to x from $x = -4$ to $x = -1$?

Choices:

(a) 5

(b) -3

(c) -5

(d) 12

(e) 6

$$\frac{f(b)-f(a)}{b-a} = \frac{f(-1)-f(-4)}{-1-(-4)} = \frac{2-17}{-1+4} = \frac{-15}{3} = -5$$

$f(-1) = -5(-1) - 3 = 5 - 3 = 2$

$f(-4) = -5(-4) - 3 = 20 - 3 = 17$

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15. Let $g(x) = \sqrt{x-4}$. Find the domain of $g(x)$.

Choices:

(a) $(-\infty, -4] \cup [4, \infty)$

(b) $[4, \infty)$

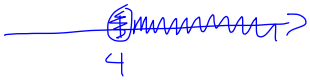
(c) All real numbers.

(d) $(-\infty, 4) \cup (4, \infty)$

(e) $(4, \infty)$

$x - 4 \geq 0$

$x \geq 4$



$[4, \infty)$

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16. Suppose the graph of $y = f(x)$ is a parabola with vertex $(-1, 3)$ and goes through the points $(0, 0)$ and $(-4, -24)$. Which of the following is an formula for $f(x)$?

Choices:

(a) $f(x) = (x-1)^2 + 3$

(b) $f(x) = x^2 + 3x$

(c) $f(x) = 2x^2 + 4x + 5$

(d) $f(x) = (x+1)(x+4)$

(e) $f(x) = -3(x+1)^2 + 3$

$y = a(x-h)^2 + k$

$y = a(x+1)^2 + 3$

$0 = a(0+1)^2 + 3$

$0 = a(1)^2 + 3$

$0 = a + 3$

$a = -3$

$y = -3(x+1)^2 + 3$

check that $(-4, -24)$ is a point on the graph

$-3(-4+1)^2 + 3$

$-3(-3)^2 + 3$

$-3(9) + 3$

$-27 + 3$

-24 ✓

---

17. Solve for x.

Choices:

(a) $x = 11$

(b) $x = -4.5$

(c) $x = \sqrt[6]{12}$

(d) $x = 0$

(e) $x = \dfrac{12}{6\log(4)}$

$\dfrac{6\log_4(x+5)}{6} = \dfrac{12}{6}$

$\log_4(x+5) = 2$

$4^2 = x + 5$

$16 = x + 5$

$-5 \quad -5$

$x = 11$

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18. Write $2\log(x) + 3\log(y) - 4\log(z)$ as a single logarithm.

Choices:

(a) $\dfrac{\log(x^2y^3)}{\log(z^4)}$

(b) $\log(x^2 + y^3 - z^4)$

(c) $\log(xyz)$

(d) $\log\left(\dfrac{2x3y}{4z}\right)$

(e) $\log\left(\dfrac{x^2y^3}{z^4}\right)$

$\log(x^2) + \log(y^3) - \log(z^4)$

$\log(x^2 \cdot y^3) - \log(z^4)$

$\log\left(\dfrac{x^2y^3}{z^4}\right)$

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19. Explain how the graph of $g(x) = (x+5)^2 - 8$ is obtained from the graph of $f(x) = x^2$.

Down 8

left 5

Choices:

(a) Shift the graph of f right 5 units and shift up 8 units to obtain the graph of g.

(b) Shift the graph of f left 8 units and shift down 5 units to obtain the graph of g.

(c) Shift the graph of f left 5 units and shift down 8 units to obtain the graph of g.

(d) Shift the graph of f right 5 units and shift down 8 units to obtain the graph of g.

(e) Shift the graph of f right 8 units and shift up 5 units to obtain the graph of g.

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20. If \$2,500 is deposited in a bank account with a yearly interest rate of 4% compounded monthly, how long until the account has doubled? Round answer to the nearest tenth.

n = 12

Choices:

(a) 10.5 years.

(b) 32.8 years.

(c) 17.4 years.

(d) 2.1 years.

(e) 21.0 years.

$$\frac{5000}{2500} = \frac{2500\left(1 + \frac{0.04}{12}\right)^{12t}}{2500}$$

$$2 = (1.00333\ldots)^{12t}$$

$$t = \frac{\ln(2)}{\ln(1.04)} = 17.6729$$

$$\frac{\ln(2)}{12\ln(1.0033\ldots)} = \frac{12t\ln(1.00333\ldots)}{12\ln(1.0033\ldots)}$$

$$t = \frac{\ln(2)}{12\ln(1.0033\ldots)} = 17.3575\ldots$$