MA109 — College Algebra Fall 2017
Final Exam 2017-12-13

Name: Key Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (c) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	(a)	(b)	(c)	(d)	(e)	12.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	13.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	14.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve: $8192^{7x-2} = 32$

$(2^{13})^{7x-2} = 2^5 \Rightarrow 2^{91x-26} = 2^5$

$\Rightarrow 91x - 26 = 5 \Rightarrow 91x = 31$

$\Rightarrow x = 31/91$

Possibilities:

(a) $\frac{31}{91}$

(b) $\frac{32}{7}$

(c) $\log(-2) - \log(7)$

(d) $\sqrt{-2} - \sqrt{7}$

(e) $\sqrt[8192]{-2} - \sqrt[32]{7}$

2. Write the domain of the function $h(x) = \log(10 - x)$ in interval notation.

$10 - x > 0 \Rightarrow -x > -10 \Rightarrow x < 10$

Possibilities:

(a) $(-\infty, 10) \cup (10, \infty)$

(b) $(-10, \infty)$

(c) $(-\infty, \infty)$

(d) $(-\infty, -10]$

(e) $(-\infty, 10)$

3. Let $f(x) = 9^x$. Which of the following is $f(-2)$?

$f(-2) = 9^{-2} = \frac{1}{9^2} = \frac{1}{81}$

Possibilities:

(a) 3

(b) $\frac{1}{9}$

(c) $\frac{1}{81}$

(d) 81

(e) $-\frac{1}{3}$

4. Solve the equation for x.

$$\log(4x+7) = \log(9x+8) + \log(10)$$

Possibilities:

(a) $x = \log\left(\frac{3}{4}\right)$ and $x = \log\left(\frac{2}{9}\right)$

(b) $x = \frac{-95 \pm \sqrt{8881}}{72}$ only

(c) No solution

(d) $x = -\frac{73}{86}$ only

(e) $x = \frac{3}{4}$ and $x = \frac{2}{9}$

$\log(4x+7) = \log((9x+8)10)$

$\Rightarrow \log(4x+7) = \log(90x+80)$

$\Rightarrow 4x+7 = 90x+80$

$\Rightarrow -86x = 73$

$\Rightarrow x = -73/86$

Domain Check: $4x+7 = 4\left(\frac{-73}{86}\right)+7 = \frac{155}{43} > 0$

$9x+8 = 9\left(\frac{-73}{86}\right)+8 = \frac{31}{86} > 0$ Answer is OK!

5. How much money should be invested at 4.25% interest, compounded monthly, so that 7 years later the investment will be worth \$25,000?

Possibilities:

(a) \$3,571.43

(b) \$18,576.58

(c) \$19,267.82

(d) \$33,455.88

(e) \$18,681.32

$A = P\left(1+\frac{r}{n}\right)^{nt} \Rightarrow 25{,}000 = P\left(1+\frac{0.0425}{12}\right)^{7\cdot 12}$

$\Rightarrow P = \frac{25{,}000}{\left(1+\frac{0.0425}{12}\right)^{84}} \approx 18{,}576.58$

6. You have already invested \$200 in a stock with an annual return of 3%. How much of an additional \$1,150 should be invested at 4% and how much at 1% so that the total return on the entire \$1,350 is 2%?

The multiple choice problem only asks for the amount at 4%.

Possibilities:

(a) \$166.67 at 4%

(b) \$216.67 at 4%

(c) \$266.67 at 4%

(d) \$316.67 at 4%

(e) \$366.67 at 4%

x: money invested at 4%

y: money invested at 1%

① $x + y = 1150$

② $200(0.03) + 0.04x + 0.01y = 1350(0.02)$

Solve ① for y: $y = 1150 - x$

Subst. into ②: $200(0.03) + 0.04x + 0.01(1150 - x) = 1350(0.02)$

$\Rightarrow 6 + 0.04x + 11.5 - 0.01x = 27 \Rightarrow 0.03x + 17.5 = 27$

$\Rightarrow 0.03x = 9.5 \Rightarrow x = \frac{9.5}{0.03} \approx 316.67$

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7. The radioactive element Fluorine-18 has a half-life of 110 minutes and is used as a medical radiotracer in PET scans. How long should it take for 79 milligrams to decay to 16 milligrams?

Possibilities:

(a) About 253.21 minutes

(b) About 253.28 minutes

(c) About 253.35 minutes

(d) About 253.42 minutes

(e) About 253.49 minutes

$A = Pe^{rt}$

Use half-life to solve for r when $P = 79$, $t = 110$, then $A = \frac{79}{2} = 39.5$:

$\frac{39.5}{79} = \frac{79e^{r \cdot 110}}{79} \Rightarrow 1/2 = e^{110r}$

$\Rightarrow \ln(1/2) = 110r \Rightarrow r = \frac{1}{110}\ln(1/2)$.

Now solve for t when $P = 79$, $A = 16$, $r = \frac{1}{110}\ln(1/2)$:

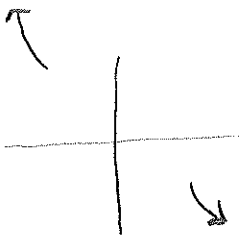
$16 = 79e^{\frac{1}{110}\ln(1/2)t} \Rightarrow 16/79 = e^{\frac{1}{110}\ln(1/2)t} \Rightarrow \ln(16/79) = \frac{1}{110}\ln(1/2)t$

$\Rightarrow 110\ln(16/79) = \ln(1/2)t \Rightarrow t = \frac{110\ln(16/79)}{\ln(1/2)} \approx 253.42$

8. Determine the end behavior of the following function.

$$(11-3x)^5$$

Leading term is $-243x^5$. It's end behavior is:



Possibilities:

(a) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(b) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(c) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(d) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(e) $y \to 0$ as $x \to \infty$ and $y \to 0$ as $x \to -\infty$

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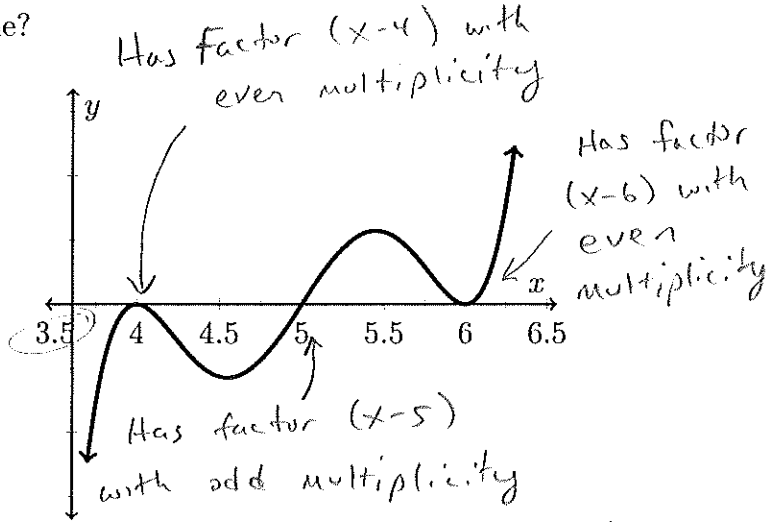
9. Which expression's graph most resembles this one?



Possibilities:

(a) $(x-4)(x-5)^2(x-6)$

(b) $(x-1)^4(x-2)^5(x-3)^6$

(c) $(x-4)^2(x-5)(x-6)^2$

(d) $(x-4)(x-5)(x-6)$

(e) $(x-1)^4(x-2)^5(x-1)^6$

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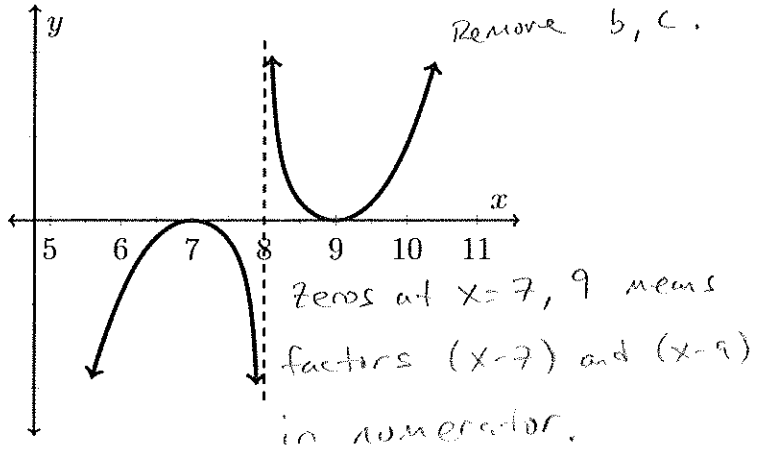
10. Which expression's graph most resembles this one?

Vertical asymptote at $x=8$ means $x-8$ in denominator. Remove b, c.

Test pt at $x=8.5$, remove a since it's negative.

Test pt at $x=7.5$, remove d since it's positive.



Possibilities:

(a) $\frac{(x-7)(x-9)}{(x-8)^2}$

(b) $\frac{(x-1)^7(x-3)^9}{(x-2)^8}$

(c) $\frac{(x-1)^7(x-1)^9}{(x-2)^8}$

(d) $\frac{(x-7)(x-9)}{(x-8)}$

(e) $\frac{(x-7)^2(x-9)^2}{(x-8)}$

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11. Let

$$f(x) = \begin{cases} 3x-1 & \text{if } x \le -2 \\ x^2+3 & \text{if } -2 < x \le 5 \\ -2x-5 & \text{if } x > 5 \end{cases}$$

Find $f(4)$.

since $-2 < 4 \le 5$!

$f(4) = 4^2 + 3 = 16 + 3 = 19$

Possibilities:

(a) 4

(b) 11

(c) 19

(d) -13

(e) 209

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12. Solve for z.

$$4z^2 - 11z + 5 = 0$$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{11 \pm \sqrt{(-11)^2 - 4(4)(5)}}{2(4)}$$

$$= \frac{11 \pm \sqrt{121-80}}{8} = \frac{11 \pm \sqrt{41}}{8}$$

Possibilities:

(a) $\frac{11 \pm \sqrt{41}}{8}$

(b) $\frac{11}{8} \pm \sqrt{101}$

(c) $\frac{-11 \pm \sqrt{41}}{8}$

(d) $\frac{-11 \pm \sqrt{201}}{8}$

(e) $\frac{11 \pm \sqrt{201}}{8}$

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13. Find an equation for the line through the points $(6,2)$ and $(3,4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-6} = \frac{2}{-3} = -2/3$$

using $y - y_0 = m(x - x_0)$:

$$y - 2 = -\frac{2}{3}(x-6)$$

Possibilities:

(a) $y = \frac{3}{2}(x-6) - 2$

(b) $y - 2 = -\frac{2}{3}(x-6)$

(c) $y + 2 = -\frac{3}{2}(x+6)$

(d) $y + 2 = -\frac{2}{3}(x+6)$

(e) $y - 2 = -\frac{3}{2}(x-6)$

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14. Which of the following statements best describes the system of equations?

$$\begin{cases} x + y = 4 & \text{①} \\ 2x + 2y = 8 & \text{②} \end{cases}$$

Mult ① by -2 and add eqns:

$$-2x - 2y = -8$$
$$+\quad 2x + 2y = 8$$
$$0 = 0 \quad \text{True!}$$

Infinite solns aka dependent.

Possibilities:

(a) The system is dependent. Every point is a solution to the system. ← Not true

(b) The system is consistent. It has exactly one solution which is $(4, 8)$. ← Not true

(c) The system is dependent. Two solutions to the system are $(1, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(0, 0)$.

(d) The system is inconsistent. Therefore the system has no solutions.

(e) The system is dependent. Two solutions to the system are $(4, 8)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$. ← Not a soln.

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15. Let $f(x) = 6x^2 + 2x - 13$. Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $\dfrac{6xh + 3h^2 - 2}{h}$

(b) $12x + 2 + 6h$

(c) $6h$

(d) 1

(e) $12x + 6h$

$$\frac{f(x+h) - f(x)}{h} = \frac{6(x+h)^2 + 2(x+h) - 13 - (6x^2 + 2x - 13)}{h}$$

$$= \frac{6(x^2 + 2xh + h^2) + 2x + 2h - 13 - 6x^2 - 2x + 13}{h}$$

$$= \frac{6x^2 + 12xh + 6h^2 + 2x + 2h - 13 - 6x^2 - 2x + 13}{h} = \frac{h(12x + 6h + 2)}{h}$$

$$= 12x + 6h + 2$$

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16. The point $(6, 2)$ is on the graph of which of the following equations?

Possibilities:

(a) $x = y - 4$

(b) $xy = 0$

(c) $4x + 12 = 4y + 12$

(d) $4x + 12 = xy + 24$

(e) $xy + 24 = xy + 8$

Plug into choice D:

$$\text{LHS} = 4x + 12 = 4(6) + 12 = 24 + 12 = 36$$

$$\text{RHS} = xy + 24 = 6(2) + 24 = 12 + 24 = 36 \checkmark$$

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17. Solve for a in $\dfrac{(7a-6)^3}{2} = 4$. $\quad 8 \cdot \dfrac{(7a-6)^3}{8} = (4)2 \Rightarrow (7a-6)^3 = 8$

Possibilities:

(a) 5324

$\Rightarrow 7a-6 = \sqrt[3]{8} \Rightarrow 7a-6 = 2 \Rightarrow 7a = 8$

(b) $\dfrac{8}{7}$ ✓

$\Rightarrow a = 8/7$

(c) $\dfrac{7}{8}$

(d) $\dfrac{26}{21}$

(e) $6 \pm \sqrt{8}/7$

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18. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = p(7238 - 700p). \qquad R(p) = -700p^2 + 7238p$$

What per-ticket price maximizes the teams total revenue?

Possibilities: Max occurs at vertex where

$p = \dfrac{-b}{2a} = \dfrac{-7238}{2(-700)} = \dfrac{7238}{1400} = 5.17$

(a) \$4.42

(b) \$4.67

(c) \$4.92

(d) \$5.17 ✓

(e) \$5.42

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19. Explain how the graph of $g(x) = \sqrt{x-9} + 1$ is obtained from the graph of $f(x) = \sqrt{x}$.

Possibilities: (Right ↑, Up ↖)

(a) Shift the graph of f right 1 units and shift up 9 units to obtain the graph of g.

(b) Shift the graph of f left 9 units and shift up 1 units to obtain the graph of g.

(c) Shift the graph of f right 9 units and shift down 1 units to obtain the graph of g.

(d) Shift the graph of f left 1 units and shift down 9 units to obtain the graph of g.

(e) Shift the graph of f right 9 units and shift up 1 units to obtain the graph of g. ✓

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20. The equation of a parabola was printed below, but ink spilled on some of the numbers. Try to answer the question anyways:

open up

$$y = 13(x - \blacksquare)^2 + \blacksquare$$

(h) (k)

Does this parabola open up or down? What is its vertex?

Possibilities:

(a) The vertex is (2,-13), but up/down can't be read

(b) The vertex is (2,13), but up/down can't be read

(c) Down, but the vertex can't be read

(d) The vertex is (13,2), but up/down can't be read

(e) Up, but the vertex can't be read

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21. A concrete walk of uniform width is to be built around a giant circular pool. The radius of the pool is 16 meters, and enough concrete is available to cover 39.84π square meters (approximately). If all the concrete is to be used, how wide should the walk be (approximately)? Choose the closest answer.

Possibilities:

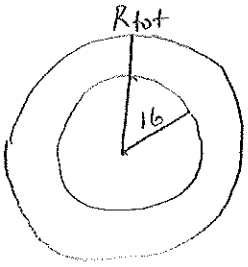
(a) 1.20 meters wide

(b) 1.37 meters wide

(c) 1.54 meters wide

(d) 1.71 meters wide

(e) 1.88 meters wide



$A_{pool} = \pi r^2 = \pi (16)^2 = 256\pi$

$A_{total} = A_{pool} + A_{conc} = 256\pi + 39.84\pi$

$= 295.84\pi$

$A_{total} = \pi R_{total}^2 \Rightarrow 295.84\pi = \pi R_{tot}^2$

$\Rightarrow R_{tot}^2 = 295.84 \Rightarrow R_{tot} = 17.2$

The concrete width is $17.2 - 16 = 1.2$ m

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22. Find all distinct, real solutions x to $(x^2 - 5)(x - 9)(x - 6) = 0$.

Possibilities:

(a) $x = \pm\sqrt{5}$, $x = -9$, and $x = -6$

(b) $x = 5$, $x = 9$, and $x = 6$

(c) $x = \pm\sqrt{5}$, $x = 9$, and $x = 6$

(d) $x = -5$, $x = -9$, and $x = -6$

(e) No solution

$x^2 - 5 = 0$ or $x - 9 = 0$ or $x - 6 = 0$

$\Rightarrow x^2 = 5$ or $x = 9$ or $x = 6$

$\Rightarrow x = \pm\sqrt{5}, 9, 6$

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Exponential Growth Model If n_0 is the initial size of a population that experiences **exponential growth**, then the population $n(t)$ at time t increases according to the model:

$$\boxed{n(t) = n_0 e^{rt}}$$

where r is the relative rate of growth of the population (expressed as a proportion of the population).

Radioactive Decay Model: If m_0 is the initial mass of a radioactive substance with half-life h, then the mass $m(t)$ remaining at time t is modeled by the function:

$$\boxed{m(t) = m_0 e^{-rt}}$$

where $r = \dfrac{\ln 2}{h}$.

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$