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The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

a b c d e

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GOOD LUCK!

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For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Write the given expression without using radicals.

$$\sqrt[11]{x^5}$$

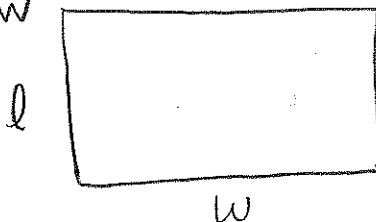
Possibilities:

- (a) x^{-6}
- (b) $x^{5/11}$
- (c) x^6
- (d) $x^5 - x^{11}$
- (e) $x^{11/5}$

$$\sqrt[11]{x^5} = (x^5)^{1/11} = x^{5/11}$$

2. A rectangle is 6 times as tall as it is wide. Express the area of the rectangle as a function of its width.

$$l = 6w$$



Possibilities:

- (a) $A(w) = 6w^2$
- (b) $A(w) = \sqrt{6w}$
- (c) $A(w) = \sqrt{\frac{w}{6}}$
- (d) $A(w) = 6w$
- (e) $A(w) = \frac{w}{6}$

$$A = (l)(w)$$

$$A = (6w)(w)$$

$$A = 6w^2$$

3. Find $f(4)$ if $f(x) = \begin{cases} 9 & \text{if } x \leq 1 \\ 2x + 7 & \text{if } 1 < x \leq 3 \\ 3x + 4 & \text{if } 3 < x \leq 5 \\ 19 & \text{if } x > 5 \end{cases} \leftarrow 3 < 4 \leq 5$

So when $x = 4$

$$f(x) = 3x + 4$$

$$\begin{aligned} f(4) &= 3(4) + 4 \\ &= 12 + 4 \\ &= 16 \end{aligned}$$

Possibilities:

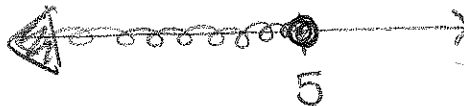
- (a) 16
- (b) 13
- (c) 19
- (d) 9
- (e) 15

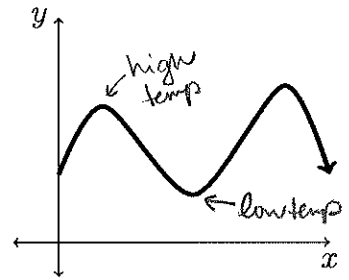
4. Find the domain of $\sqrt{5-x} + \sqrt{6}$

Possibilities:

- (a) $(5, \infty)$
- (b) $(-\infty, 5]$
- (c) $(-\infty, \infty)$
- (d) $[5, \infty)$
- (e) $(-\infty, 5) \cup (6, \infty)$

$$\begin{aligned} 5 - x &\geq 0 \\ 5 &\geq x \end{aligned}$$





5. Which situation below is most reasonably depicted in this graph:

Possibilities:

- (a) y is the number of bacteria at time x if the bacteria experience a steady rate of exponential growth.
- (b) y is the temperature of left-over food at time x if the food is placed in the refrigerator at time $x = 0$.
- (c) y is the distance from home at time x as you run to the end of the block and back at a steady pace.
- (d) y is the outside temperature after x hours, if $x = 0$ is the morning of the first day.
- (e) y is the amount of water in a bucket at time x if a hole is made in the bucket at time $x = 0$.

6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 10 to 25 seconds.

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	50	200	450	800	1250	1800

Possibilities:

- (a) 80 feet per second
- (b) 60 feet per second
- (c) 70 feet per second
- (d) 50 feet per second
- (e) 20 feet per second

$$\begin{aligned}
 \text{avg rate} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{1250 - 200}{25 - 10} \\
 &= 70
 \end{aligned}$$

7. Simplify the formula for the average rate of change of $f(x) = (x-2)^2 + 9$ from $x = 6$ to $x = 6+h$

Possibilities:

- (a) $8+h$
- (b) $2h$
- (c) h
- (d) 1
- (e) $12+h$

$$\begin{aligned} \text{ARoC} &= \frac{f(6+h) - f(6)}{(6+h) - 6} \\ &= \frac{[(6+h-2)^2 + 9] - [(6-2)^2 + 9]}{h} \\ &= \frac{(h+4)^2 + 9 - 25}{h} = \frac{h^2 + 8h + 16 + 9 - 25}{h} \\ &= \frac{h^2 + 8h}{h} = \frac{h(h+8)}{h} \\ &= \boxed{h+8} \end{aligned}$$

8. Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 2x^2 + 9x + 6$ and $g(x) = 7x - 8$

Possibilities:

- (a) $(-\infty, \infty)$
- (b) $(-\infty, \frac{8}{7}) \cup (\frac{8}{7}, \infty)$
- (c) $[\frac{8}{7}, \infty)$
- (d) $(-\infty, \frac{7}{8})$
- (e) $\left[\frac{-9 \pm \sqrt{9^2 - 4(2)(6)}}{4}, \infty\right)$

$$\left(\frac{f}{g}\right)(x) = \frac{2x^2 + 9x + 6}{7x - 8}$$

$$7x - 8 \neq 0$$

$$7x \neq 8$$

$$x \neq \frac{8}{7}$$

9. Find the rule of the function $(f \circ g)(x)$ if $f(x) = \frac{1}{5x+7}$ and $g(x) = x^{17} + 19$

Possibilities:

(a) x

(b) $\frac{7x-5}{19x-17}$

(c) $\frac{1}{5(x^{17}+19)+7}$

(d) $\left(\frac{1}{5x+7}\right)^{17} + 19$

(e) $5(\sqrt[17]{x-19}) - 7$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x^{17}+19) \\ &= \frac{1}{5(x^{17}+19)+7}\end{aligned}$$

10. Find $(f-g)(4)$ where $f(x) = 3x^2 - 6x - 9$ and $g(x) = 2x - 5$

Possibilities:

(a) 12

(b) 25

(c) 2

(d) 18

(e) 0

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= 3x^2 - 6x - 9 - [2x - 5] \\ &= 3x^2 - 8x - 4\end{aligned}$$

$$\begin{aligned}(f-g)(4) &= 3(4)^2 - 8(4) - 4 \\ &= 12\end{aligned}$$

11. A certain fungus grows in a circular shape. Its diameter in inches after t weeks is given below.

$$9 - 5e^{-3t} = d$$

Which of the following is an expression for the area covered as a function of time?

Possibilities:

(a) $A(t) = \pi 81 - 25e^{-9t}$

(b) $D(t) = 9 - 5e^{-3t}$

(c) $A(t) = \pi t^2$

(d) $t = \ln(5/9)/3$

(e) $A(t) = \pi \left(\frac{9 - 5e^{-3t}}{2} \right)^2$

$$A = \pi r^2$$

and $r = \frac{d}{2} = \frac{9 - 5e^{-3t}}{2}$

So $A = \pi \left(\frac{9 - 5e^{-3t}}{2} \right)^2$

12. Suppose that the graph of $y = f(x)$ contains the point $(9, 5)$. Which of these points must be on the graph of $y = g(x)$ for $g(x) = 3 + 4f(x + 8)$?

$$\Rightarrow f(9) = 5$$

Possibilities:

(a) $(6, 17)$

(b) $\left(17, \frac{1}{2}\right)$

(c) $\left(\frac{1}{4}, 8\right)$

(d) $(1, 23)$

(e) $(12, 2)$

we need $x+8=9$
 since 9 is the only input into f we know
 thus $x+8=9$
 $x=1$

$$\begin{aligned} \text{then } g(1) &= 3 + 4f(1+8) \\ &= 3 + 4f(9) \\ &= 3 + 4(5) \\ &= 3 + 20 \\ &= 23 \end{aligned}$$

$$g(1) = 23 \Rightarrow (1, 23)$$

-
13. Which sequence of transformations will transform the graph of the function f into the graph of the function g ?

$$f(x) = \sqrt{x} + 5 \quad g(x) = \sqrt{x-3} + 6$$

Possibilities:

- (a) shift left by 3 then shift down by 1
- (b) shift left by 3 then shift up by 1
- (c) shift right by 3 then shift down by 1
- (d) shift left by 1 then shift down by 3
- (e) shift right by 3 then shift up by 1

to change f to g ,

we need to "subtract 3 on the inside" of the square root \Rightarrow shift right 3

we also need to "add 1 on the outside" to make 5 into 6 \Rightarrow shift up 1

-
14. Use algebra to find the inverse of the given one-to-one function.

$$f(x) = (x^6 + 4)^5$$

$$y = (x^6 + 4)^5$$

switch x and y terms

$$x = (y^6 + 4)^5$$

solve for y

$$\sqrt[5]{x} = y^6 + 4$$

$$\sqrt[5]{x} - 4 = y^6$$

$$\sqrt[6]{\sqrt[5]{x} - 4} = y$$

Possibilities:

- (a) $f^{-1}(x) = x^{30} + 4$
- (b) $f^{-1}(x) = \sqrt[6]{\sqrt[5]{x} - 5}$
- (c) $f^{-1}(x) = \sqrt[5]{\sqrt[6]{x} - 4}$
- (d) $f^{-1}(x) = (x^5 + 4)^6$
- (e) $f^{-1}(x) = \sqrt[6]{\sqrt[5]{x} - 4}$

15. Use algebra to find the inverse of the given one-to-one function. $f(x) = \frac{9x}{5x+3}$

Possibilities:

(a) $f^{-1}(x) = \frac{5x+3}{9x}$

(b) $f^{-1}(x) = \frac{9x}{5x-3}$

(c) $f^{-1}(x) = \frac{9}{5}x + 3$

(d) $f^{-1}(x) = \frac{3x}{9-5x}$

(e) $f^{-1}(x) = \frac{3x}{9x+5}$

$$y = \frac{9x}{5x+3}$$

Switch x and y

$$x = \frac{9y}{5y+3}$$

Solve for y

$$(5y+3)x = 9y$$

$$5yx + 3x = 9y$$

$$3x = 9y - 5yx$$

$$3x = y(9-5x) \rightarrow y = \frac{3x}{9-5x}$$

16. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week: 1 2 3 4 5 6

Frogs: 60 180 540 1620 4860 14580

$\xrightarrow{\times 3} \xrightarrow{\times 3} \xrightarrow{\times 3} \xrightarrow{\times 3} \xrightarrow{\times 3}$ multiplied by 3 $\Rightarrow a=3$

Which exponential growth model most closely matches the observations, if t is the week number?

Possibilities:

(a) $3(60^{(t/7)})$

(b) $3(60^t)$

(c) $20(3^t)$

(d) $20(9^{(t/7)})$

(e) $60(9^t)$

$$P(t) = P_0 a^t$$

$$P(t) = P_0 (3)^t \quad \text{since } a=3$$

We know $P(1) = 60$

$$60 = P_0 (3)^1$$

$$60 = P_0 \cdot 3$$

$$20 = P_0$$

thus $P(t) = 20(3)^t$

-
17. Determine how much money will be in a savings account if the initial deposit was \$120 and the interest rate is 3.93% compounded continuously for 2 years, 5 months. (Round your answer to the nearest cent.)

Possibilities:

- (a) \$131.72
- (b) \$131.96
- (c) \$132.20
- (d) \$132.44
- (e) \$132.68

$$\begin{aligned} P(t) &= P_0 e^{rt} \\ &= 120 e^{(.0393)(2\frac{5}{12})} \\ &= 120 e^{(.0393)(2.416\bar{6})} \\ &= 131.9557637 \\ &= \$131.96 \end{aligned}$$

-
18. Translate the given logarithmic statement into an equivalent exponential one.

$$\begin{array}{l} \log_9(5x+3) = 14 \\ \uparrow \text{base} \qquad \leftarrow \text{resulting expression} \\ \qquad \qquad \qquad \leftarrow \text{exponent} \end{array}$$
$$9^{14} = 5x+3$$

Possibilities:

- (a) $(5x+3)^{14} = 9$
- (b) $(5x+3)^9 = 14$
- (c) $(9)^{5x+3} = 14$
- (d) $(9)^{14} = 5x+3$
- (e) $(14)^{5x+3} = 9$

19. Write the domain of the function $h(x) = \log(x - 17)$ in interval notation.

Possibilities:

- (a) $(-\infty, 17]$
- (b) $(-\infty, 17) \cup (17, \infty)$
- (c) $(-\infty, -17)$
- (d) $(-\infty, \infty)$
- (e) $(17, \infty)$

$$x - 17 > 0$$

$$x > 17$$

20. Write the given expression as a single logarithm.

$$9 \log(x) + \log(5y) - \log(3z)$$

Possibilities:

- (a) $\log\left(\frac{x^9 y^5}{z^3}\right)$
- (b) $\log(x^9 y^5 z^3)$
- (c) $\log\left(\frac{x^9(5y)}{3z}\right)$
- (d) $\log(9x + 5y - 3z)$
- (e) $\log(9x(5 + y) - 3 - z)$

$$= \log(x^9) + \log(5y) - \log(3z)$$

$$= \log(x^9 \cdot 5y) - \log(3z)$$

$$= \log\left(\frac{x^9 5y}{3z}\right)$$

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt} \quad (\text{if compounded } n \text{ times per year})$$

$$P(t) = P_0 e^{rt} \quad (\text{if compounded continuously}).$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

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