

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (c) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e)  | 11. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e)  | 12. (a) (b) (c) (d) (e) |
| 3. (a) (b) (c) (d) (e)  | 13. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e)  | 14. (a) (b) (c) (d) (e) |
| 5. (a) (b) (c) (d) (e)  | 15. (a) (b) (c) (d) (e) |
| 6. (a) (b) (c) (d) (e)  | 16. (a) (b) (c) (d) (e) |
| 7. (a) (b) (c) (d) (e)  | 17. (a) (b) (c) (d) (e) |
| 8. (a) (b) (c) (d) (e)  | 18. (a) (b) (c) (d) (e) |
| 9. (a) (b) (c) (d) (e)  | 19. (a) (b) (c) (d) (e) |
| 10. (a) (b) (c) (d) (e) | 20. (a) (b) (c) (d) (e) |

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

**Multiple Choice Questions**

Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.

1. Find all distinct real solutions  $x$  to

$$4(6x + 8) = 7x + 9$$

$$24x + 32 = 7x + 9 \implies 17x = -23$$

Possibilities:

- (a) 0
- (b)  $\frac{1}{17}$
- (c)  $-17 \pm \sqrt{19}$
- (d)  $-\frac{23}{17}$
- (e)  $-\frac{23}{24}$

$$\implies x = \frac{-23}{17}$$

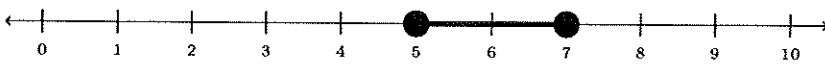
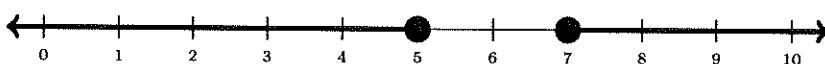
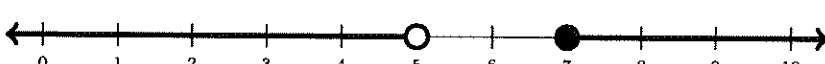
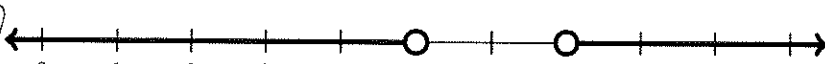
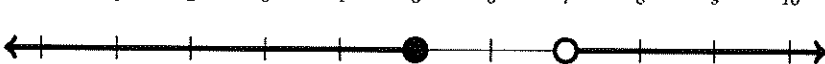
2. What is the first operation applied to  $x$  in the following expression?  $4 - (6x + 8)^7$

Possibilities:

- (a) Add 8
- (b) Raise it to the 7th power
- (c) Multiply by 6
- (d) Take the 7th root
- (e) Subtract it from 4

3. Which of the following number lines represents the union of intervals  $(-\infty, 5) \cup (7, \infty)$

Possibilities:

- (a) 
- (b) 
- (c) 
- (d) 
- (e) 

4. Simplify, and write the given number without using absolute values.  $|\sqrt{5} - 7|$

Possibilities:

- (a)  $7 - \sqrt{5}$
- (b)  $\sqrt{5} - 7$
- (c)  $-7 - \sqrt{5}$
- (d)  $7 + \sqrt{5}$
- (e) 44

5. Find the distance between  $-\frac{6}{11}$  and 4

Possibilities:

- (a)  $\frac{50}{11}$
- (b)  $-\frac{38}{11}$
- (c)  $\frac{19}{11}$
- (d)  $\frac{38}{11}$
- (e)  $\frac{25}{11}$

$$\left| -\frac{6}{11} - 4 \right| = \left| -\frac{6}{11} - \frac{44}{11} \right| = \left| -\frac{50}{11} \right| = \frac{50}{11}$$

6. Solve the equation for  $x$ .

$$A = \pi(R+x)^2 + (L+4)(W+2) - 8$$

Possibilities:

- (a)  $x = \pm\sqrt{8}$
- (b)  $x = \frac{-R \pm \sqrt{(L+4)(W+2) - 8}}{2\pi}$
- (c)  $x = \frac{\pm\sqrt{A} \pm \sqrt{(L+4)(W+2) - 8}}{\pi} - R$
- (d)  $x = \pm\sqrt{\frac{A - (L+4)(W+2) + 8}{\pi}} - R$
- (e)  $x = \pm\sqrt{A - \pi R^2 - (L+4)(W+2) + 8}$

$$A + 8 - (L+4)(W+2) = \pi(R+x)^2$$

$$\Rightarrow (R+x)^2 = \frac{A + 8 - (L+4)(W+2)}{\pi}$$

$$\Rightarrow R+x = \pm\sqrt{\frac{A + 8 - (L+4)(W+2)}{\pi}}$$

$$\Rightarrow x = -R \pm \sqrt{\frac{A + 8 - (L+4)(W+2)}{\pi}}$$

7. Solve the equation.  $(x + 8)^4 - 57 = 24$

Possibilities:

(a) 4039 and -4153

**(b) -5 and -11**

(c) 1048519 and -1048633

(d) 59 and 55

(e) 4039 and -4039

$$\begin{aligned} (x+8)^4 - 57 = 24 &\Rightarrow (x+8)^4 = 81 \\ \Rightarrow x+8 = \pm \sqrt[4]{81} &\Rightarrow x+8 = \pm 3 \\ \Rightarrow x = -8 \pm 3 & \\ \Rightarrow x = -5, -11 & \end{aligned}$$

8. Solve for  $x$  in the equation  $|4 - 6x| = -4 + 2x$

Possibilities:

(a) 0 only

(b) 1 and 0

**(c) No real solutions.**

(d)  $\frac{1}{2}$  only

(e) 1 only

$$\begin{aligned} 4 - 6x = -4 + 2x &\quad \text{or} \quad 4 - 6x = -(-4 + 2x) \\ \Rightarrow 8 = 8x &\quad \Rightarrow 4 - 6x = 4 - 2x \\ \Rightarrow x = 1 &\quad \Rightarrow -4x = 0 \\ &\quad \Rightarrow x = 0 \end{aligned}$$

check  $x=0$ : LHS =  $|4-0|=4$ ; RHS =  $-4+0=-4$  No!

check  $x=1$ : LHS =  $|4-6|=|-2|=2$ ; RHS =  $-4+2=-2$  No!

9. Solve for  $x$  in the equation  $\sqrt{11x + 37} = x + 5$

Possibilities:

(a) -3 only

(b) 1 only

(c) 4 only

**(d) -3 and 4**

(e) No real solutions.

$$\begin{aligned} (\sqrt{11x+37})^2 &= (x+5)^2 \Rightarrow 11x+37 = x^2+10x+25 \\ \Rightarrow x^2 - x - 12 &= 0 \Rightarrow (x-4)(x+3) = 0 \\ \Rightarrow x = 4, -3 & \end{aligned}$$

check  $x=4$ : LHS =  $\sqrt{44+37} = \sqrt{81} = 9$ ; RHS =  $4+5=9$  yes!

check  $x=-3$ : LHS =  $\sqrt{-33+37} = \sqrt{4} = 2$ ; RHS =  $-3+5=2$  yes!

10. Solve for  $x$  in  $\frac{5}{x-7} + \frac{12}{x-6} = \frac{8}{(x-7)(x-6)}$

Possibilities:

(a)  $\frac{8}{5}$  and  $\frac{2}{3}$

(b) 7 and 6

(c)  $\frac{8}{17}$  only

(d)  $\sqrt{13}$  and  $-\sqrt{13}$

(e)  $\frac{122}{17}$  only

$$(x-7)(x-6) \left[ \frac{5}{x-7} + \frac{12}{x-6} \right] = \left[ \frac{8}{(x-7)(x-6)} \right] (x-7)(x-6)$$

$$\Rightarrow 5(x-6) + 12(x-7) = 8$$

$$\Rightarrow 5x - 30 + 12x - 84 = 8$$

$$\Rightarrow 17x - 114 = 8 \Rightarrow 17x = 122$$

$$\Rightarrow x = \frac{122}{17}$$

11. Find all distinct, real solutions to

$$\frac{x+5}{x+7} + \frac{x+9}{x+6} = 3$$

Possibilities:

(a)  $x = 4 \pm \sqrt{6}$

(b)  $x = -8$  and  $x = -\frac{9}{2}$

(c)  $x = -2$  and  $x = -6$

(d) No real solutions

(e)  $x = -6 \pm \sqrt{3}$

$$(x+7)(x+6) \left[ \frac{x+5}{x+7} + \frac{x+9}{x+6} \right] = 3 \left[ (x+7)(x+6) \right]$$

$$\Rightarrow (x+5)(x+6) + (x+9)(x+7) = 3(x+7)(x+6)$$

$$\Rightarrow x^2 + 11x + 30 + x^2 + 16x + 63 = 3(x^2 + 13x + 42)$$

$$\Rightarrow 2x^2 + 27x + 93 = 3x^2 + 39x + 126$$

$$\Rightarrow 0 = x^2 + 12x + 33 \Rightarrow x = \frac{-12 \pm \sqrt{12^2 - 4(1)(33)}}{2(1)}$$

$$\Rightarrow x = \frac{-12 \pm \sqrt{144 - 132}}{2} = \frac{-12 \pm \sqrt{12}}{2} = \frac{-12 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow x = -6 \pm \sqrt{3}$$

12. Solve for  $x$  by completing the square in  $x^2 + 2\pi x - 7 = 0$

Possibilities:

(a)  $\sqrt{7-\pi}$

(b)  $7-\pi$

(c)  $\frac{7}{1+\pi}$

(d)  $\frac{7 \pm \sqrt{19^2 - \pi}}{2}$

(e)  $-\pi \pm \sqrt{\pi^2 + 7}$

$$x^2 + 2\pi x + \pi^2 = 7 + \pi^2$$

$$\Rightarrow (x + \pi)(x + \pi) = 7 + \pi^2$$

$$\Rightarrow (x + \pi)^2 = 7 + \pi^2 \Rightarrow x + \pi = \pm \sqrt{7 + \pi^2}$$

$$\Rightarrow x = -\pi \pm \sqrt{7 + \pi^2}$$

13. Find a number  $k$  such that the equation  $x^2 + kx + 7 = 0$  has exactly one real solution.

Possibilities:

(a)  $\frac{\pm\sqrt{7}}{2}$

(b)  $\pm\sqrt{7}$

(c)  $\frac{49}{4}$

(d) 49

(e)  $\pm 2\sqrt{7}$

$$b^2 - 4ac = 0 \Rightarrow k^2 - 4(1)(7) = 0$$

$$\Rightarrow k^2 - 28 = 0 \Rightarrow k^2 = 28 \Rightarrow k = \pm\sqrt{28}$$

$$\Rightarrow k = \pm\sqrt{4 \cdot 7} \Rightarrow k = \pm 2\sqrt{7}$$

14. Find all distinct, real solutions  $x$  to  $x^6 - 14x^3 - 11 = 0$ .

Hint: You may want to complete the square, or simplify a root/fraction before finishing the problem.

$$\text{Let } u = x^3: \quad u^2 - 14u - 11 = 0$$

Possibilities:

(a)  $\pm\sqrt[3]{7 \pm \sqrt{60}}$

(b)  $\sqrt[3]{7 \pm \sqrt{60}}$

(c)  $\pm\sqrt[3]{7 \pm \sqrt[3]{60}}$

(d)  $\pm\sqrt{7 \pm \sqrt[3]{60}}$

(e)  $\pm\sqrt{7 + \sqrt[3]{60}}$

$$\Rightarrow u = \frac{14 \pm \sqrt{(-14)^2 - 4(1)(-11)}}{2(1)} = \frac{14 \pm \sqrt{196 + 44}}{2}$$

$$\Rightarrow u = \frac{14 \pm \sqrt{240}}{2} = \frac{14 \pm \sqrt{4 \cdot 60}}{2} = \frac{14 \pm 2\sqrt{60}}{2}$$

$$\Rightarrow u = 7 \pm \sqrt{60} \Rightarrow x^3 = 7 \pm \sqrt{60}$$

$$\Rightarrow x = \sqrt[3]{7 \pm \sqrt{60}}$$

15. Find all distinct, real solutions  $x$  to  $(x^2 - 5)(x - 2)(x - 8) = 0$ .

Possibilities:

(a)  $x = \pm\sqrt{5}, x = 2, \text{ and } x = 8$

(b)  $x = 5, x = 2, \text{ and } x = 8$

(c)  $x = -5, x = -2, \text{ and } x = -8$

(d)  $x = \pm\sqrt{5}, x = -2, \text{ and } x = -8$

(e) No solution

$$x^2 - 5 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{or} \quad x - 8 = 0$$

$$\Rightarrow x^2 = 5 \qquad \qquad \qquad \Rightarrow x = 2 \qquad \qquad \qquad \Rightarrow x = 8$$

$$\Rightarrow x = \pm\sqrt{5}$$

16. Find an equation for the circle shown below:

Possibilities:

(a)  $(x + 5)^2 + (y - 6)^2 = 9$

(b)  $(x - 5)^2 + (y + 6)^2 = 9$

(c)  $(x + 5)^2 + (y + 6)^2 = 3$

(d)  $(x - 10)^2 + (y - 12)^2 = -9$

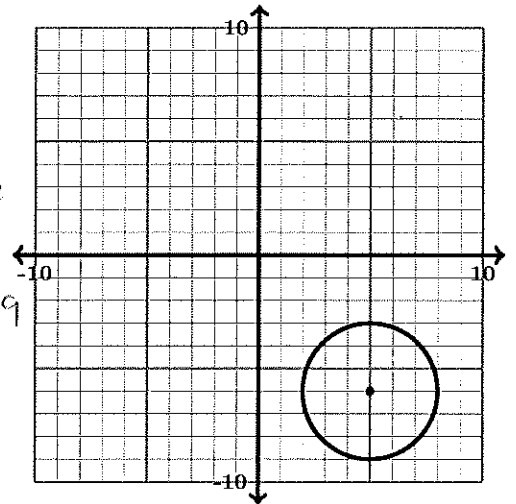
(e)  $(x - 5)^2 + (y - 6)^2 = 3$

Center:  $(5, -6)$

Radius: 3

$$(x - 5)^2 + (y - (-6))^2 = 3^2$$

$$\Rightarrow (x - 5)^2 + (y + 6)^2 = 9$$



17. The graph of  $x^2 + y^2 - 16x - 18y + 120 = 0$  is a circle. Find its center and its radius.

Possibilities:

(a) Radius: 10 Center:  $(16, 18)$

(b) Radius: 5 Center:  $(-8, -9)$

(c) Radius: 5 Center:  $(8, 9)$

(d) Radius:  $2\sqrt{30}$  Center:  $(-8, -9)$

(e) Radius:  $2\sqrt{30}$  Center:  $(8, 9)$

$$x^2 - 16x + 64 + y^2 - 18y + 81 = -120 + 64 + 81$$

$$\Rightarrow (x - 8)^2 + (y - 9)^2 = 25$$

Center:  $(8, 9)$

Radius:  $\sqrt{25} = 5$

18. What is the distance between (4, 6) and (9, 5)?

Possibilities:

(a)  $\sqrt{6}$

(b)  $\sqrt{26}$

(c) 5

(d) 1

(e)  $\sqrt{290}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9 - 4)^2 + (5 - 6)^2}$$
$$= \sqrt{5^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26}$$

19. Find the slope of the line in the graph.

Possibilities:

(a) 5

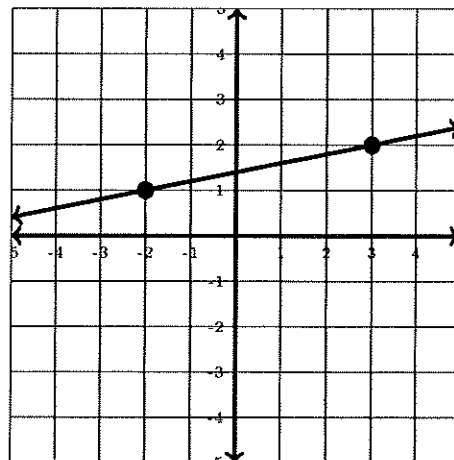
(b)  $-\frac{1}{5}$

(c)  $\frac{1}{5}$

(d) -5

(e) The slope is not defined.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{3 - -2}$$
$$= \frac{1}{3 + 2} = \frac{1}{5}$$



20. Find an equation for the line through the points (4, 6) and (8, 7).

Possibilities:

(a)  $y - 6 = \frac{1}{4}(x - 4)$

(b)  $y - 6 = 4(x - 4)$

(c)  $y + 6 = \frac{1}{4}(x + 4)$

(d)  $y = -4(x - 4) - 6$

(e)  $y + 6 = 4(x + 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 6}{8 - 4} = \frac{1}{4}$$
$$y - y_0 = \frac{1}{4}(x - x_0) \Rightarrow y - 6 = \frac{1}{4}(x - 4)$$
$$\Rightarrow y - 6 = \frac{1}{4}x - 1 \Rightarrow y = \frac{1}{4}x + 5$$