7 A Strategy for Application Problems

Concepts:

- Solving Application Problems

(Section 2.3)

Although there is no hard and fast algorithm for solving application problems, there are strategies for approaching application problems that can help you to understand the problem, organize your thoughts, solve the problem, and communicate your solution.

A Strategy for Application Problems:

1. **READ**: Use a dictionary if necessary.
2. **DEFINE UNKNOWNS**: Use variables and describe them with words and/or a picture.
3. **DESCRIBE RELATIONSHIPS WITH MATHEMATICAL SYMBOLS**
4. **SIMPLIFY TO ONE EQUATION**
5. **SOLVE THE EQUATION**
6. **ANSWER THE QUESTION**: The answer may not be the variable you solved for.

Example 7.1 (Number 1 from Section 2.3 in your textbook)
A student has exam scores of 88, 62, and 79. What score does he need on the fourth exam to have an average of 80?

Let x be the student's fourth exam score.

$$\frac{88+62+79+x}{4} = 80$$

$$\frac{229+x}{4} = 80$$

$$229+x = 320$$

$$x = 91$$

The student needs a 91.

Example 7.2 (Similar to Number 2 from Section 2.3 in your textbook)
How many gallons of a 12% acid solution should be combined with 10 gallons of an 18% acid solution to obtain a 16% acid solution?

Note:
concentration = $\frac{\text{gallons pure acid}}{\text{total gallons of solution}}$

(pure acid in 18% sol'n) + (pure acid in 12% sol'n) = (pure acid in 16% sol'n)

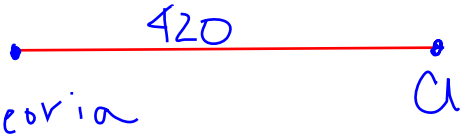
$.18(10) + .12x = .16(10+x)$

$1.8 + .12x = 1.6 + .16x$

$0.2 = .04x \Rightarrow x = 5$

5 gallons of 12% acid should be added.

Example 7.3 (Example 8 from Section 2.3 in your textbook)
A pilot wants to make a the 840-mile round trip from Cleveland to Peoria and back in 5 hours flying time. Going to Peoria, there will be a headwind of 30 mph, that is, a wind opposite to the direction the plane is flying. It is estimated that on the return trip to Cleveland, there will be a 40-mph tailwind (in the direction the plane is flying). At what constant speed should the plane be flown?



Recall distance = rate · time

$\text{time} = \frac{\text{distance}}{\text{rate}}$

Let x be the plane's constant speed.

Time to fly C→P + Time to fly P→C = 5

$\frac{420}{x-30} + \frac{420}{x+40} = 5$

$420(x+40) + 420(x-30) = 5(x+40)(x-30)$

$420x + 16800 + 420x - 12600 = 5x^2 + 50x - 6000$

$0 = 5x^2 - 790x - 10{,}200$

$x = \frac{790 \pm \sqrt{790^2 - 4(5)(-10{,}200)}}{2(5)}$

$= \frac{790 \pm 910}{10}$

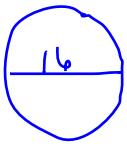
$= \frac{790+910}{10}$ or $\frac{790-910}{10}$

$= 170$ or -12

negative so it doesn't make sense

The plane's constant speed is 170 mph

Example 7.4 (Number 7 from Section 2.3 of your textbook)
The diameter of a circle is 16 cm. By what amount must the radius be decreased to decrease the area by 48π square centimeters?



Current Area: $\pi r^2 = \pi(8)^2 = 64\pi$

New Area: $64\pi - 48\pi = 16\pi = \pi(4)^2$

New Radius: 4cm

current radius: 8cm

The current radius must be decreased by $8-4=4$cm.

Example 7.5 (Number 6 from Section 2.3 in your textbook)
A merchant has 5 pounds of mixed nutes that cost \$30. He wants to add peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 50 pounds of a mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?

	Current mix	Peanuts	Cashews	New Mix
		p	c	50 lbs
Weight	5 lbs			
Total Cost	\$30	1.50p	4.50c	2.90(50)=145
cost/lbs		\$1.50	\$4.50	\$2.90

$5 + p + c = 50$ $\quad p = 45 - c$

Total Cost of New Mix = Total Cost of current mix + Total Cost of peanuts + Total Cost cashews

$145 = 30 + 1.5p + 4.5c$

$145 = 30 + 1.5(45-c) + 4.5c$

$145 = 30 + 67.5 - 1.5c + 4.5c$

$47.5 = 3c$

$15.83 \approx c$

$p = 45 - c \approx 45 - 15.83$

≈ 29.17

29.17 lbs peanuts

7.1 A Strategy for Application Practice Problems

1. (Number 11 from Section 2.3 of your textbook) A radiator contains 8 quarts of fluid, 40% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 60% antifreeze?

2. (Number 19 from Section 2.3 of your textbook) A 13 foot long ladder leans on a wall. The bottom of the ladder is 5 feet from the wall. If the bottom is pulled out 3 feet farther from the wall, how far does the top of the ladder move down the wall?

3. A chemist has two large containers of hydrochloric acid(HCl) solution. The concentration of the acid is different in the two containers. She blends 150mL of the first solution with 50mL of the second solution to obtain a solution that is 14.750% acid. She blends 350mL of the first solution with 50mL of the second solution to obtain a solution that is 15.875% acid. What are the concentrations of hydrochloric acid in the original containers?

4. (Number 25 from Section 2.3 of your textbook) Red Riding Hood drives the 432 miles to Grandmother's house in 1 hour less than it takes the Wolf to drive the same route. Her average speed is 6 mph faster than the Wolf's average speed. How fast does each drive?

5. (Number 15 from Section 2.3 of your textbook) The average of two numbers is 41.125, and their product is 1683. What are the numbers?

6. (Number 26 from Section 2.3 in your textbook) To get to work, Sam jogs 3 kilometers to the train and then rides the remaining 5 kilometers. If the train goes 40 kilometers per hour faster than Sam's constant rate of jogging and the entire trip takes 30 minutes, how fast does Sam jog?