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(Sections 11.1, 11.1A)

Solutions to Systems of Equations

A **system of equations** is a set of equations involving the same variables. For example,

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 4 \end{aligned} \quad (1)$$

is a system of equations in the variables x and y.

Solutions to Systems of Equations

A **solution of a system** is a solution of ALL the equations in the system at the same time. Often times when a system involves the variables x and y, we write the solution as a point.

Solutions to Systems of Equations

Example 1

Verify that $(3, -1)$ is a solution to the system

$$\begin{aligned} 2x + y &= 5 \\ x - y &= 4 \end{aligned}$$

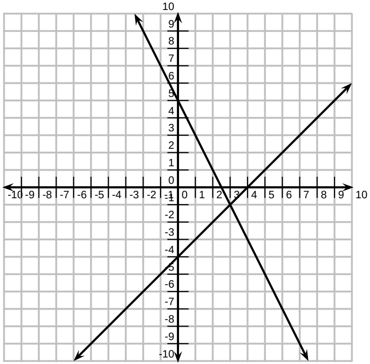
Solutions to Systems of Equations

Recall that we visualize the set of solutions to a single equation in two variables as a graph. We visualize a solution to a system of equations as the set of points that are on the graphs of all the equations in the system at the same time. In other words, we visualize a solution to a system of equations as ________________________________.

Solutions to Systems of Equations

Example 2

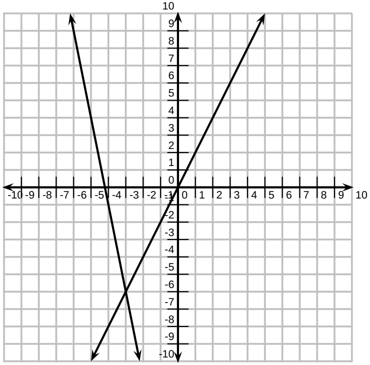
Below are the graphs of the equations $2x + y = 5$ and $x - y = 4$. Notice the two graphs intersect at the point $(3, -1)$, which is also the solution to the system. (Can you tell which graph is which?)



Solutions to Systems of Equations

Example 3 (Do you understand solutions to systems?)

Below are the graphs of the equations $4x - 2y = 0$ and $21 + y = -5x$. (Can you tell which graph is which?)



Solutions to Systems of Equations

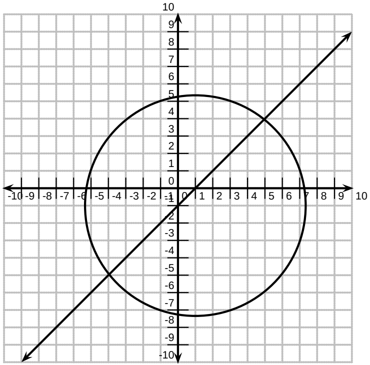
How many solutions does the system of equations below have? Use the graph to find approximate solution(s). Verify your solution(s) algebraically.

$$\begin{aligned} 4x &- 2y &= 0 \\ 21 &+ y &= -5x. \end{aligned}$$

Solutions to Systems of Equations

Example 4 (Do you understand solutions to systems?)

Below are the graphs of the equations $(x-1)^2 + (y+1)^2 = 41$ and $x - y = 1$.



Solutions to Systems of Equations

How many solutions does the system of equations below have? Use the graph to find approximate solution(s). Verify your solution(s) algebraically.

$$\begin{aligned} (x-1)^2 + (y+1)^2 &= 41 \\ x - y &= 1. \end{aligned}$$

Solutions to Systems of Equations

Example 5 (Do you understand solutions to systems?)

Suppose you have a system of equations where the graphs of both equations are lines. How many solutions could you possibly have?

(Note: Systems of equations in which every equation in the system is linear are called *linear systems*.)

Solutions to Systems of Equations

It is useful to know how to visualize solutions to a system. However, the only way to guarantee an **exact** solution is to solve the equation **algebraically**. Solutions that are obtained by **graphical** means are **approximations**.

Solving Systems of Equations - Substitution Method

Example 6 (Substitution Method)

Given the system of equations,

$$\begin{array}{rcrcr} 4x & - & 2y & = & 0 \\ 21 & + & y & = & -5x \end{array}$$

we know that any solution to this system must satisfy the second equation. Solve this system of equations by solving the second equation for y and substituting the result in for y in the first equation.

Solving Systems of Equations - Substitution Method

The Substitution Method

1. Solve one equation for one the variables.
2. Substitute it into the OTHER equation.
3. Once you have a value for one of your variables, substitute this value into one of your original equations to solve for the other variable.

Solving Systems of Equations - Substitution Method

Example 7 (Substitution Method)

Use the substitution method to solve the system of equations below. Express your solution as a point.

$$\begin{array}{rcrcr} 2x & - & y & = & 1 \\ 3x & + & 2y & = & 4 \end{array}$$

Solving Systems of Equations - Substitution Method

Example 8 (Substitution Method)

Use the substitution method to solve the system of equations below.

$$\begin{aligned} (x-1)^2 + (y+1)^2 &= 41 \\ x - y &= 1. \end{aligned}$$

How many solutions does the system have?

Solving Systems of Equations - Substitution Method

Example 9 (Substitution Method)

Use the substitution method to solve the system of equations below.

$$\begin{array}{rcrcr} 3x & - & 6y & = & 9 \\ 2x & - & 4y & = & 6 \end{array}$$

How many solutions does the system have? What are three solutions to the system?

Solving Systems of Equations - Substitution Method

When a system of equations involves two equations that are equivalent to each other, we call that system **dependent**.

Solving Systems of Equations - Elimination Method

Recall that when you add or multiply the same quantity to both sides of an equation, the result is an equivalent equation. Equivalent equations are equations with exactly the same solution set.

Solving Systems of Equations - Elimination Method

Example 10 (Elimination Method)

Given the system of equations,

$$\begin{array}{rcrcr} 5x & - & 3y & = & 0 \\ 4x & - & 6y & = & -6 \end{array}$$

solve the system by adding a multiple of the first equation to the second equation in order to eliminate the y variable.

Solving Systems of Equations - Elimination Method

The Elimination Method

1. Adjust the coefficients of each equation so that the coefficients of one of the variables are additive inverses.
2. Add the two equations together to eliminate one of the variables.
3. Solve for the non-eliminated variable.
4. Once you have a value for one of your variables, substitute this value into one of your original equations to solve for the other variable.

Solving Systems of Equations - Elimination Method

Example 11 (Elimination Method)

Use the elimination method to solve the system of equations below.

$$\begin{array}{rcrcr} 2x & - & y & = & 1 \\ 3x & + & 2y & = & 4 \end{array}$$

Solving Systems of Equations - Elimination Method

Example 12 (Elimination Method)

Use the elimination method to solve the system of equations below.

$$\begin{aligned} x^3 + 4y^2 &= 12 \\ x + y^2 &= 3. \end{aligned}$$

How many solutions does the system have?

Solving Systems of Equations - Elimination Method

Example 13 (Elimination Method)

Use the elimination method to solve the system of equations below.

$$\begin{array}{rcrcr} 12x & - & 6y & = & 0 \\ 10x & - & 5y & = & 15 \end{array}$$

How many solutions does the system have?

Solving Systems of Equations - Elimination Method

Systems of equations in which there are no solutions are called **inconsistent**. An example of an inconsistent system is one where the graphs of the equations are parallel lines. Parallel lines never intersect and thus, there is no solution to the system of equations.

Solving Systems of Equations

Example 14 (Points of Intersection)

Find the points of intersection between the graphs of $2x + y - 4 = 0$ and $(x + 1)^2 + y^2 = 9$. Which method should you use to solve this system?

Approximate Solutions of Equations in One Variable

Remember, the only way to guarantee an **exact** solution is to solve the equation **algebraically**. Solutions that are obtained by **graphical** means are **approximations**.

Approximate Solutions of Equations in One Variable

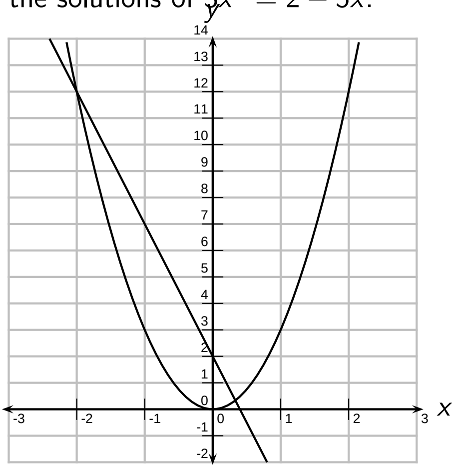
Example 15 (The Intersection and Intercept Methods)

(a) Solve the equation.

$$3x^2 = 2 - 5x$$

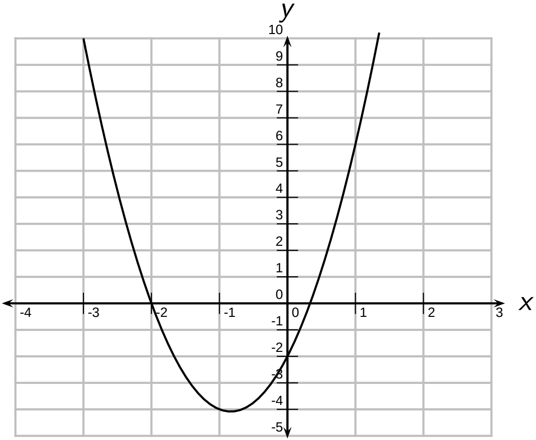
Approximate Solutions of Equations in One Variable

(b) (*The Intersection Method*) The graphs of $y = 3x^2$ and $y = 2 - 5x$ are shown below. Explain how these graphs can be used to approximate the solutions of $3x^2 = 2 - 5x$.



Approximate Solutions of Equations in One Variable

(c) (*The Intercept Method*) The graph of $y = 3x^2 + 5x - 2$ is shown below. Explain how this graphs can be used to approximate the solutions of $3x^2 = 2 - 5x$.



Approximate Solutions of Equations in One Variable

(d) Explain how the Intercept Method is related to the Intersection Method.