MA109 — College Algebra Fall 2016
Final Exam 2016-12-15

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	c	d	**e**	12.	a	b	c	**d**	e
2.	a	b	c	d	**e**	13.	**a**	b	c	d	e
3.	a	b	c	d	**e**	14.	**a**	b	c	d	e
4.	a	**b**	c	d	e	15.	a	b	c	d	**e**
5.	a	**b**	c	d	e	16.	a	b	**c**	d	e
6.	a	b	c	**d**	e	17.	a	b	**c**	d	e
7.	a	b	**c**	d	e	18.	**a**	b	c	d	e
8.	**a**	b	c	d	e	19.	a	**b**	c	d	e
9.	a	b	**c**	d	e	20.	a	b	c	**d**	e
10.	a	**b**	c	d	e	21.	**a**	b	c	d	e
11.	a	b	c	**d**	e	22.	a	b	c	**d**	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

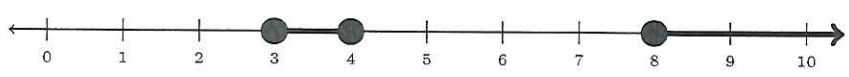
Name: _____

Multiple Choice Questions

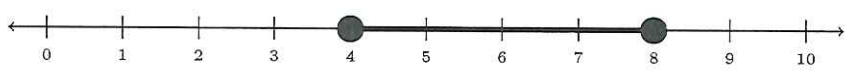
Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

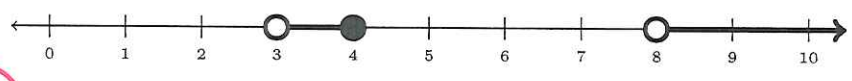
1. Which of the following number lines represents the union of intervals $[3, 4) \cup [8, \infty)$?

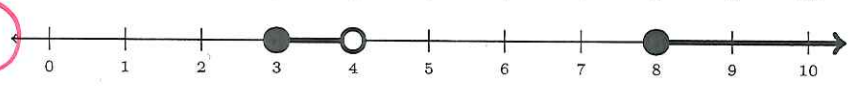
Possibilities:

(a) 

(b) 

(c) 

(d) 

(e) 

2. Solve for r in:

$$(r^2 - 25)(2r - 14) = 0$$

$r^2 - 25 = 0$ $\quad$ $2r - 14 = 0$

$r^2 = 25$ $\quad$ $2r = 14$

$r = \pm\sqrt{25}$ $\quad$ $r = 7$

$r = \pm 5$

Possibilities:

(a) The only real solutions are ± 5.
(b) The only real solutions are 2 and 25.
(c) The only real solutions are 14 and 25.
(d) The only real solutions are 2 and 0.
(e) The only real solutions are 7 and ± 5.

3. The point $(5, 2)$ is on the graph of which of the following equations?

Possibilities:

(a) $xy = 0$
(b) $x = y - 3$
(c) $4x + 10 = 4y + 10$
(d) $xy + 20 = xy + 8$
(e) $4x + 10 = xy + 20$

a) $(5)(2) = 10 \neq 0$

b) $5 \stackrel{?}{=} 2 - 3$

$5 \neq -1$

c) $4(5) + 10 \stackrel{?}{=} 4(2) + 10$

$20 + 10 \neq 8 + 10$

d) $(5)(2) + 20 \stackrel{?}{=} (5)(2) + 8$

$10 + 20 \neq 10 + 8$

e) $4(5) + 10 \stackrel{?}{=} (5)(2) + 20$

$20 + 10 \stackrel{?}{=} 10 + 20$

$30 = 30$

4. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -2 \\ x^2 + 3 & \text{if } -2 < x \leq 5 \\ -2x - 5 & \text{if } x > 5 \end{cases}$$

Find $f(10)$.

Possibilities:

(a) 29

(b) -25

(c) 19

(d) 103

(e) -76

$10 > 5$

$f(10) = -2(10) - 5$

$= -20 - 5$

$= -25$

5. Solve for z.

$$3z^2 - 11z + 9 = 0$$

Possibilities:

(a) $\frac{11}{6} \pm \sqrt{94}$

(b) $\frac{11 \pm \sqrt{13}}{6}$

(c) $\frac{-11 \pm \sqrt{229}}{6}$

(d) $\frac{11 \pm \sqrt{229}}{6}$

(e) $\frac{-11 \pm \sqrt{13}}{6}$

$z = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(9)}}{2(3)}$

$= \frac{11 \pm \sqrt{121 - 108}}{6}$

$= \frac{11 \pm \sqrt{13}}{6}$

6. Solve: $4096^{23x-3} = 32$

Possibilities:

(a) $\log(-3) - \log(23)$

(b) $\sqrt{-3} - \sqrt{23}$

(c) $\sqrt[4096]{-3} - \sqrt[32]{23}$

(d) $\frac{41}{276}$

(e) $\frac{32}{23}$

$23x-3$

$4096 = 32$

$(2^{12})^{23x-3} = 2^5$

$2^{12(23x-3)} = 2^5$ ← equal bases must have equal exponents!

$12(23x-3) = 5$

$276x - 36 = 5$

$276x = 41$

$x = 41/276$

$2^{12} = 4096$

$2^5 = 32$

---

7. The mass M of a radioactive element at time T is given by the equation below, where c is the original mass and h is the half-life of the element. The half-life of a substance is 6.2 years. How long will it take for 175 grams to decay to 35 grams?

c ↑ 175 grams

$M(T)$ ↑ 35 grams

h ↑ 6.2 years

$M(T) = c\left(.5^{T/h}\right)$

$35 = 175(.5^{T/6.2})$

Possibilities:

(a) About 14.0 years

(b) About 14.2 years

(c) About 14.4 years

(d) About 14.6 years

(e) About 14.8 years

$\frac{35}{175} = .5^{T/6.2}$

$\log\left(\frac{35}{175}\right) = \log\left(.5^{T/6.2}\right)$

$\log\left(\frac{35}{175}\right) = \frac{T}{6.2}\log(.5)$

$6.2\log\left(\frac{35}{175}\right) = T\log(.5)$

$\frac{6.2\log\left(\frac{35}{175}\right)}{\log(.5)} = T \approx 14.39595$ years

8. Solve the equation.

$$\log(x-3)+\log(x-2)=\log(9x-39)$$

Possibilities:

(a) $x=5$ and $x=9$

(b) $x=-2$ and $x=-1$

(c) $x=\frac{34}{7}$ only

(d) $x=3$ and $x=2$

(e) $x=-4$ and $x=-8$

$\log[(x-3)(x-2)]=\log(9x-39)$ ← equal base logs must have equal arguments!

$x^2-5x+6=9x-39$

$x^2-14x+45=0$

$(x-9)(x-5)=0$

$x-9=0 \Rightarrow \boxed{x=9}$ $\quad x-5=0 \Rightarrow \boxed{x=5}$

---

9. Write the domain of the function $h(x)=\log(x-10)$ in interval notation.

Possibilities:

(a) $(-\infty,\infty)$

(b) $(-\infty,10)\cup(10,\infty)$

(c) $(10,\infty)$

(d) $(-\infty,-10)$

(e) $(-\infty,10]$

↑ must be positive!

$x-10>0$

$x>10$

10 $\boxed{(10,\infty)}$

---

10. Suppose the graph of $y=f(x)$ is a parabola with vertex $(-1,3)$ and goes through the point $(0,5)$. Which of the following is a formula for $f(x)$?

Possibilities:

(a) $f(x)=2(x+1)^2+5$

(b) $f(x)=2(x+1)^2+3$

(c) $f(x)=2(x-1)^2+3$

(d) $f(x)=2(x-1)(x-3)$

(e) $f(x)=2(x+1)(x+5)$

$f(x)=a(x-h)^2+k$ (h k; x f(x)) $\Rightarrow \boxed{f(x)=2(x+1)^2+3}$

$5=a(0+1)^2+3$

$5=a(1)+3$

$2=a$

---

11. The graph of a polynomial function is shown. List each root of the polynomial in increasing order and state whether its multiplicity is even or odd.

Possibilities:

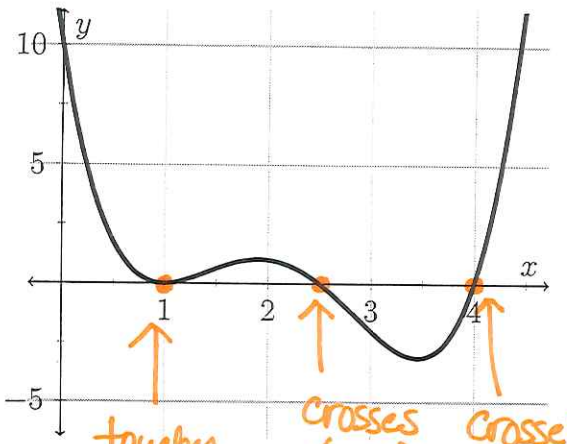
(a) 2.0 (even) and 3.5 (odd)

(b) 1.0 (odd), 2.5 (even), and 4.0 (even)

(c) 2.5 (odd) and 4.0 (even)

(d) 1.0 (even), 2.5 (odd), and 4.0 (odd)

(e) 1.0 (odd) only



touches (even) 1

crosses (odd) 2.5

crosses (odd) 4

12. The graph of a polynomial function is shown. What does the graph suggest about the degree and leading coefficient?

Possibilities:

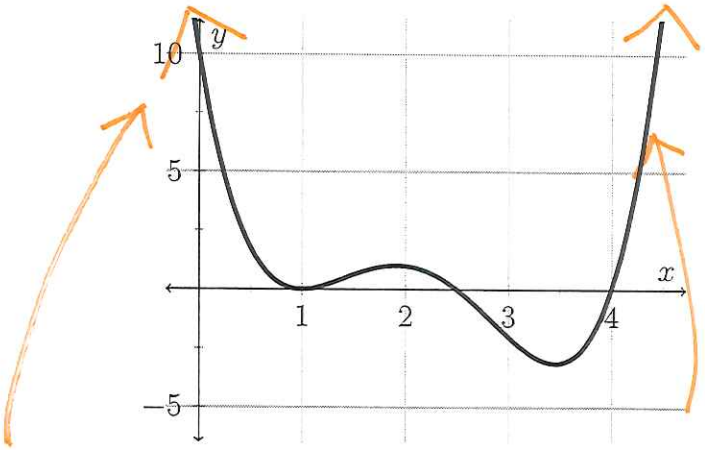
(a) Degree is odd, leading coefficient is negative

(b) Degree is odd, leading coefficient is positive

(c) Degree is even, leading coefficient is negative

(d) Degree is even, leading coefficient is positive

(e) Degree is zero, leading coefficient is zero



$y \to \infty$ as $x \to -\infty$

$y \to \infty$ as $x \to \infty$

* even degree

* $a_n > 0$

13. Factor the polynomial as a product of linear factors and a factor $g(x)$ such that $g(x)$ is either a constant or a polynomial that has no rational roots.

$$x^4 - 13x^3 + 25x^2 - 39x + 66$$

*possible roots are factors of 66 $\Rightarrow \pm1, \pm2, \pm3, \pm6, \pm11, \pm22, \pm33, \pm66$

$P(1) \neq 0$ | $P(2) = 0$ | $P(3) \neq 0$

$P(6) \neq 0$ | $P(11) = 0$ | $P(22) \neq 0$

$P(33) \neq 0$ | $P(66) \neq 0$

Possibilities:

(a) $(x-2)(x-11)(x^2+3)$

(b) $(x+1)(x+2)(x+11)(x-13)$

(c) $(x-1)(x-2)(x-3)(x-11)$

(d) $(x-1)(x-2)(x-7)(x-13)$

(e) $(x-2)(x-13)(x^2+7)$

$(x-2)(x-11) \rightarrow x^2 - 13x + 22$

x^2+3 | $(x-2)$ & $(x-11)$ must be factors!

$x^4 - 13x^3 + 25x^2 - 39x + 66$

$-(x^4 - 13x^3 + 22x^2)$

$3x^2 - 39x + 66$

$3x^2 - 39x + 66$

use long division to find remaining factor

has no rational roots!

14. Analyze the function algebraically. List its vertical asymptotes, y-intercept, and horizontal asymptote, if any.

$$\frac{7x^2}{(x+2)(x-11)}$$

VA: come from zeros in denominator, NOT also zeros in numerator

$x = -2$ & $x = 11$

Possibilities:

(a) Vertical asymptotes at $x = -2$ and $x = 11$,
y-intercept at $y = 0$, and
horizontal asymptote at $y = 7$

(b) Vertical asymptotes at $x = 2$ and $x = 11$,
y-intercept at $y = \frac{7}{(2)(-11)}$, and
no horizontal asymptote

(c) Vertical asymptotes at $x = 0$ and $x = -7$,
y-intercept at $y = 2$, and
horizontal asymptote at $y = 11$

(d) Vertical asymptotes at $x = 0$ and $x = 7$,
y-intercept at $y = -2$, and
horizontal asymptote at $y = 11$

(e) Vertical asymptotes at $x = -11$ and $x = 2$,
y-intercept at $y = 7$, and
horizontal asymptote at $y = 0$

*y-intercept happens when $x = 0$

$\frac{7(0)^2}{(0+2)(0-11)} = 0$ ↑ y-int.

HA: ratio of leading terms

$\frac{7x^2}{x^2} \Rightarrow 7$

↑

$y = 7$

15. The equation of a parabola was printed below, but ink spilled on some of the numbers. Try to answer the question anyways:

$$y = 23(x - \blacksquare)^2 + \blacksquare$$

Does this parabola open up or down? What is its vertex?

Possibilities:

(a) Down, but the vertex can't be read

(b) The vertex is (2,23), but up/down can't be read

(c) The vertex is (2,-23), but up/down can't be read

(d) The vertex is (23,2), but up/down can't be read

(e) Up, but the vertex can't be read

h k ⟹ (h,k) vertex

a>0 opens up

23>0

16. A large apple orchard is planning how many trees to plant. They have so many acres, they just record the average number of trees per acre. When there are an average of 23.5 apple trees per acre, the average yield has been found to be 480 apples per tree. For each additional tree planted per acre, the yield per tree decreases by 16 apples per tree. How many additional trees per acre should be planted to maximize the yield? Round to the nearest 0.25 trees per acre.

Possibilities:

(a) About 2.75 additional trees per acre

(b) About 3.00 additional trees per acre

(c) About 3.25 additional trees per acre

(d) About 3.50 additional trees per acre

(e) About 3.75 additional trees per acre

x = # of additional trees

yield is (# of trees/acre)(# of apples/tree)

$(23.5 + x)(480 - 16x)$

$-16x^2 + 104x + 11280$

max yield happens at vertex

$h = \frac{-b}{2a} \Rightarrow \frac{-104}{2(-16)} = 3.25$

Conditions for achieving max yield

17. Find an equation for the line through the points $(3,5)$ and $(4,9)$.

Possibilities:

(a) $y-5=\frac{1}{4}(x-3)$

(b) $y+5=\frac{1}{4}(x+3)$

(c) $y-5=4(x-3)$

(d) $y=-\frac{1}{4}(x-3)-5$

(e) $y+5=4(x+3)$

$x_1\ x_2$

$m=\frac{9-5}{4-3}=\frac{4}{1}=4$

point slope form!

$y-y_1=m(x-x_1)$

$\boxed{y-5=4(x-3)}$

---

18. Which of the following statements best describes the system of equations?

$$\begin{cases} x+y=4 \\ x+2y=8 \end{cases}$$

$(x+y=4)\cdot -1 \rightarrow -x-y=-4$

$x+2y=8$

$y=4$

Possibilities:

(a) The system is consistent. It has exactly one solution which is $(0,4)$.

(b) The system is inconsistent. Therefore the system has no solutions.

(c) The system is dependent. Two solutions to the system are $(4,8)$ and $(2,2)$. One point that is NOT a solution to the system is $(1,1)$.

(d) The system is dependent. Two solutions to the system are $(1,3)$ and $(0,4)$. One point that is NOT a solution to the system is $(0,0)$.

(e) The system is consistent. It has exactly one solution which is $(4,8)$.

$x+y=4$

$x+4=4$

$x=0$

$\boxed{\text{Our Solution} \Rightarrow (0,4)}$

19. You have already invested \$300 in a stock with an annual return of 10%. How much of an additional \$1,350 should be invested at 20% and how much at 5% so that the total return on the entire \$1,650 is 15%?

x = amount @ 20% y = amount @ 5%

The multiple choice problem only asks for the amount at 20%.

Possibilities:

(a) \$600 at 20%

(b) \$1000 at 20%

(c) \$1,100 at 20%

(d) \$550 at 20%

(e) \$350 at 20%

Amounts $\Rightarrow$ $300 + x + y = 1650$

Investment Value $\Rightarrow$ $(.10)(300) + .20x + .05y = (.15)(1650)$

Using substitution method $\Rightarrow y = 1350 - x$

$30 + .20x + .05(1350 - x) = 247.50$

$.20x + 67.5 - .05x = 247.50 - 30$

$.15x = 217.50 - 67.5$

$x = \frac{150}{.15} = \boxed{1000}$

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20. Let $f(x) = 5x^2 + 2x - 17$. Find $\frac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $5h$

(b) 1

(c) $\frac{6xh + 3h^2 - 2}{h}$

(d) $10x + 2 + 5h$

(e) $10x + 5h$

$f(x+h) = 5(x+h)^2 + 2(x+h) - 17$

$= 5(x^2 + 2xh + h^2) + 2x + 2h - 17$

$= 5x^2 + 10xh + 5h^2 + 2x + 2h - 17$

$$\frac{(5x^2 + 10xh + 5h^2 + 2x + 2h - 17) - (5x^2 + 2x - 17)}{h}$$

$$\frac{10xh + 5h^2 + 2h}{h} \Rightarrow \frac{h(10x + 5h + 2)}{h}$$

---

21. Solve for b in $\dfrac{(6b-5)^3}{3} = 9$.

Possibilities:

(a) $\frac{4}{3}$

(b) $\frac{117649}{3}$

(c) $\frac{7}{3}$

(d) $\frac{3}{4}$

(e) $5 \pm \sqrt{27}/6$

$$\frac{(6b-5)^3}{3} = 9$$

$$(6b-5)^3 = 27$$

$$6b-5 = \sqrt[3]{27}$$

$$6b = 3+5$$

$$b = \frac{8}{6}$$

$$\boxed{b = \frac{4}{3}}$$

22. Explain how the graph of $g(x) = \sqrt{x-9}+4$ is obtained from the graph of $f(x) = \sqrt{x}$.

shifts 4 units up

shifts right 9 units

Possibilities:

(a) Shift the graph of f right 9 units and shift down 4 units to obtain the graph of g.

(b) Shift the graph of f left 4 units and shift down 9 units to obtain the graph of g.

(c) Shift the graph of f right 4 units and shift up 9 units to obtain the graph of g.

(d) Shift the graph of f right 9 units and shift up 4 units to obtain the graph of g.

(e) Shift the graph of f left 9 units and shift up 4 units to obtain the graph of g.

$f(x) = \sqrt{x}$

$f(x-9) = \sqrt{x-9}$

$f(x-9)+4 = \sqrt{x-9}+4 = g(x)$

affects input → horizontal

affects output → vertical

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Exponential Growth Model If n_0 is the initial size of a population that experiences **exponential growth**, then the population $n(t)$ at time t increases according to the model:

$$\boxed{n(t) = n_0 e^{rt}}$$

where r is the relative rate of growth of the population (expressed as a proportion of the population).

Radioactive Decay Model: If m_0 is the initial mass of a radioactive substance with half-life h, then the mass $m(t)$ remaining at time t is modeled by the function:

$$\boxed{m(t) = m_0 e^{-rt}}$$

where $r = \dfrac{\ln 2}{h}$.

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$