MA109 — College Algebra Fall 2016
Exam 3 2016-11-16

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	11.	a	b	c	**d**	e
2.	a	b	c	**d**	e	12.	a	b	c	d	**e**
3.	**a**	b	c	d	e	13.	a	b	c	d	**e**
4.	a	b	c	**d**	e	14.	a	b	**c**	d	e
5.	a	**b**	c	d	e	15.	a	b	**c**	d	e
6.	a	b	**c**	d	e	16.	a	**b**	c	d	e
7.	**a**	b	c	d	e	17.	a	b	c	**d**	e
8.	a	b	c	d	**e**	18.	**a**	b	c	d	e
9.	a	b	**c**	d	e	19.	a	**b**	c	d	e
10.	a	b	c	d	**e**	20.	a	**b**	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Formulas

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

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1. Find $f(2)$ if $f(x) = \begin{cases} 9 & \text{if } x \leq 1 \\ 2x+7 & \text{if } 1 < x \leq 3 \\ 3x+4 & \text{if } 3 < x \leq 5 \\ 19 & \text{if } x > 5 \end{cases}$ ← $1 < 2 \leq 3$ (2 = input)

Possibilities:

(a) 11

(b) 13

(c) 10

(d) 19

(e) 9

$f(2) = 2(2) + 7$

$f(2) = 4 + 7$

$f(2) = 11$

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2. Find the indicated value of the function when $x = \sqrt{2} + 7$.

$$f(x) = \sqrt{x+9} - x - 6$$

$f(\sqrt{2}+7) =$

Possibilities:

(a) $\sqrt{\sqrt{2}+16} - \sqrt{2} - 1$

(b) 3

(c) $\sqrt{16} - 13$

(d) $\sqrt{\sqrt{2}+16} - \sqrt{2} - 13$

(e) $\sqrt{18} - \sqrt{2} - 13$

$f(\sqrt{2}+7) = \sqrt{(\sqrt{2}+7)+9} - (\sqrt{2}+7) - 6$

$= \sqrt{\sqrt{2}+7+9} - \sqrt{2} - 7 - 6$

$= \sqrt{\sqrt{2}+16} - \sqrt{2} - 13$

3. Find the domain of $\dfrac{5}{\sqrt{x-1}}$

Possibilities:

(a) $(1, \infty)$

(b) $(-\infty, 1) \cup (1, \infty)$

(c) $[1, \infty)$

(d) $(-\infty, \infty)$

(e) $[\frac{5}{1}, \infty)$

$x - 1 > 0$

$x > 1$

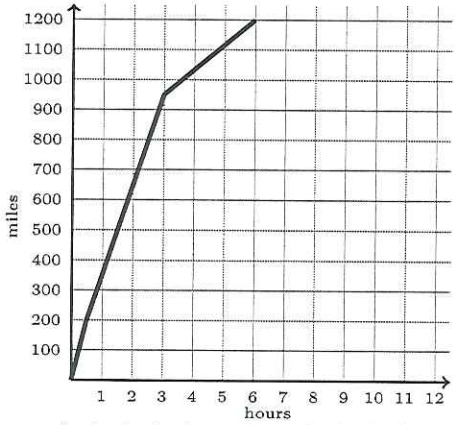
$(1, \infty)$

4. A plane flies from Austin, Texas, to Cleveland, Ohio, a distance of 1200 miles. Bad weather forces the plane to land in Dallas (about 200 miles from Austin), remain overnight (for 7.5 hours), and continue the next day.

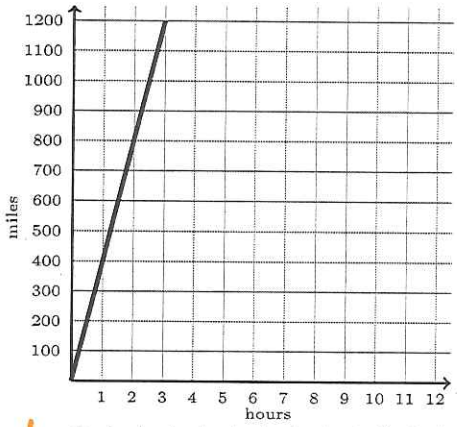
   Let f be the function whose rule is $f(t)$ is the distance (in miles) from Austin at time t hours with domain the times between initial takeoff from Austin at $t = 0$ and the final landing in Cleveland. Determine the most plausible graph of f under the given circumstance.

Possibilities:

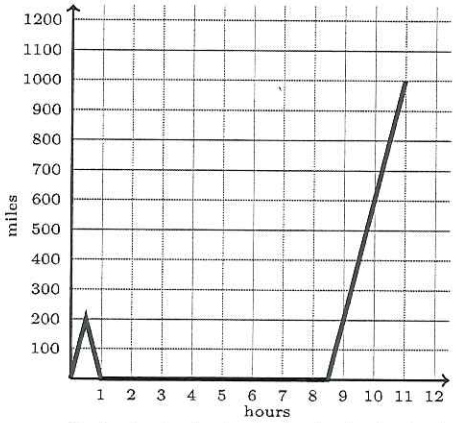
(a)



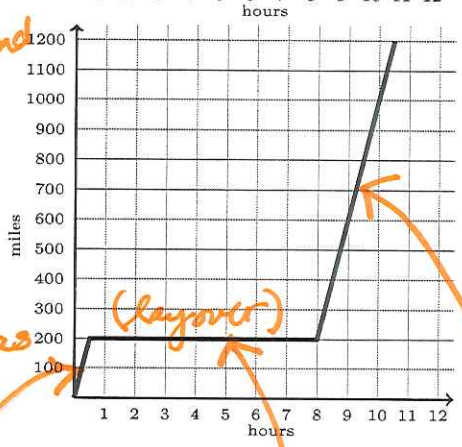
(b)



(c)



(d)

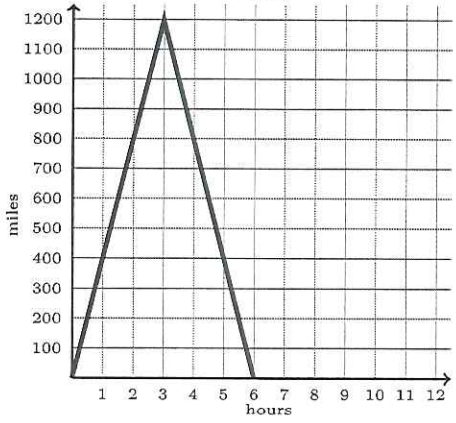


cleveland

Dallas

(layover)

(e)



* the distance from Austin is increasing for the first 200 miles
* the distance from Austin remains constant during the layover in Dallas
* the distance from Austin increases again (after the layover) until they reach Cleveland

5. Find the domain of $\sqrt[3]{\frac{x-5}{2}} + \log(9)$

Possibilities:

(a) $(-\infty, 5) \cup (9, \infty)$

(b) $(-\infty, \infty)$

(c) $(5, \infty)$

(d) $[\frac{5}{2}, \infty)$

(e) $[5, \infty)$

* they only variable is in the numerator of a fraction & under an odd root... both of which require No domain restrictions.

6. The table shows the federal income tax rates for a single person.

Taxable Income		
Over	But not over	Tax
\$0	\$5,100	14% of income
\$5,100	\$23,000	\$714 + 27% of amount over \$5,100
\$23,000		\$5,547 + 32% of amount over \$23,000

$x \leq 5100 \Rightarrow$ TAX = .14x

$5100 < x \leq 23000$

$x > 23000$

Write the rule of a piecewise-defined function T such that $T(x)$ is the tax due on a taxable income of x dollars.

$714 + .27(x - 5100)$ ← salary over \$5100

$5547 + .32(x - 23000)$ ← Salary over \$23,000

Possibilities:

(a) $T(x) = \begin{cases} 5100x & \text{if } x \leq 5100 \\ 23000x & \text{if } 5100 < x \leq 23000 \\ 5547x & \text{if } x > 23000 \end{cases}$

(b) $T(x) = \begin{cases} 5100x - 0.14 & \text{if } x \leq 5100 \\ 23000x - 0.27 & \text{if } 5100 < x \leq 23000 \\ 5547x - 0.32 & \text{if } x > 23000 \end{cases}$

(c) $T(x) = \begin{cases} 0.14x & \text{if } x \leq 5100 \\ 714.00 + 0.27(x - 5100) & \text{if } 5100 < x \leq 23000 \\ 5547.00 + 0.32(x - 23000) & \text{if } x > 23000 \end{cases}$

(d) $T(x) = \begin{cases} 0.14x & \text{if } x \leq 5100 \\ 714.00 + 0.27x & \text{if } 5100 < x \leq 23000 \\ 5547.00 + 0.32x & \text{if } x > 23000 \end{cases}$

(e) $T(x) = \begin{cases} 0.14x & \text{if } x \leq 5100 \\ 0.27x & \text{if } 5100 < x \leq 23000 \\ 0.32x & \text{if } x > 23000 \end{cases}$

7. Simplify the formula for the average rate of change of $f(x) = (x-5)^2 + 1$ from $x = 5$ to $x = 5+h$

Possibilities:

(a) h

(b) $5 + 2h$

(c) $2h$

(d) 1

(e) $10 + h$

a b

$*A.R.O.C. \Rightarrow \dfrac{f(b)-f(a)}{b-a}$

$$\frac{f(5+h)-f(5)}{(5+h)-5} = \frac{[(5+h)-5]^2+1-[(5-5)^2+1]}{h}$$

$$= \frac{h^2+1-1}{h}$$

$$= \frac{h\cdot h}{h}$$

$$= \boxed{h}$$

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8. Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 2x^2 + 9x + 4$ and $g(x) = 3x - 7$

Possibilities:

(a) $(-\infty, \frac{3}{7})$

(b) $[\frac{7}{3}, \infty)$

(c) $(-\infty, \infty)$

(d) $\left[\frac{-9\pm\sqrt{9^2-4(2)(4)}}{4}, \infty\right)$

(e) $(-\infty, \frac{7}{3}) \cup (\frac{7}{3}, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{2x^2+9x+4}{3x-7}$$ ← denominator cannot $= 0$!!

$3x-7=0$

$3x=7$

$x=7/3$ ← EXCLUDE this Value!

$7/3$

$(-\infty, 7/3) \cup (7/3, \infty)$

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9. Find $(f \circ g)(2)$ where $f(x) = |3x - 5|$ and $g(x) = 4x - 9$

Possibilities:

(a) -2

(b) 56

(c) 8

(d) 46

(e) 2

$(f \circ g)(2) = f(g(2))$

$g(2) = 4(2) - 9$
$= 8 - 9$
$= -1$

$= f(-1)$

$= |3(-1) - 5|$

$= |-3 - 5|$

$= |-8| \Longrightarrow \boxed{8}$

10. Simplify the formula for $(f \circ g)(x)$ if $f(x) = 3 - x$ and $g(x) = \dfrac{3x - 9}{x}$

Hint: try plugging in $x = 23$

Possibilities:

(a) $\dfrac{3}{x}$

(b) x

(c) $23x$

(d) $\dfrac{1}{x}$

(e) $\dfrac{9}{x}$

$(f \circ g)(x) = f(g(x))$

$= f\left(\dfrac{3x-9}{x}\right)$

$= 3 - \dfrac{3x-9}{x}$

$= 3 \cdot \dfrac{x}{x} - \dfrac{3x-9}{x}$

$= \dfrac{3x - (3x-9)}{x}$

$= \dfrac{9}{x}$

USING HINT:

$(f \circ g)(23) = f(g(23))$

$= f\left(\dfrac{3 \cdot 23 - 9}{23}\right)$

$= f\left(\dfrac{60}{23}\right)$

$= 3 - \dfrac{60}{23}$

$= \dfrac{3 \cdot 23 - 60}{23}$

$= \dfrac{69 - 60}{23}$

$= \dfrac{9}{23}$

11. Suppose that the graph of $y = f(x)$ contains the point $(9, 6)$. Find a point that must be on the graph of $y = g(x)$ for $g(x) = 2 + f(7x + 3)$.

Possibilities:

(a) $\left(\frac{6}{7}, 4\right)$

(b) $\left(-\frac{12}{7}, 4\right)$

(c) $(66, 8)$

(d) $\left(\frac{6}{7}, 8\right)$

(e) $(66, 4)$

f(9)

New input!

$9 = 7x + 3$

$6 = 7x$

$\frac{6}{7} = x$

New output

$g(\frac{6}{7}) = 2 + f(7 \cdot \frac{6}{7} + 3)$

$= 2 + f(6 + 3)$

$= 2 + f(9)$

$= 2 + 6$

$= 8$

$(\frac{6}{7}, 8)$

12. Which sequence of transformations will transform the graph of the function f into the graph of the function g?

$f(x) = \log(x) + 4 \qquad g(x) = \log(x - 3) + 6$

shifts right 3 units

shifts up 2 units

Possibilities:

(a) shift right by 3 then shift down by 2

(b) shift left by 2 then shift down by 3

(c) shift left by 3 then shift down by 2

(d) shift left by 3 then shift up by 2

(e) shift right by 3 then shift up by 2

13. Use algebra to find the inverse of the given one-to-one function.

$$f(x) = 8 + \sqrt[9]{4x-6}$$

Possibilities:

(a) $f^{-1}(x) = \dfrac{(x-9)^8+4}{6}$

(b) $f^{-1}(x) = \sqrt[9]{4x+2}$

(c) $f^{-1}(x) = \sqrt[9]{4x+6} - 8$

(d) $f^{-1}(x) = 9x^4 + 2$

(e) $f^{-1}(x) = \dfrac{(x-8)^9+6}{4}$

$y = 8 + \sqrt[9]{4x-6}$

$x = 8 + \sqrt[9]{4y-6}$ ← switch variables

$x - 8 = \sqrt[9]{4y-6}$

$(x-8)^9 = 4y - 6$

$(x-8)^9 + 6 = 4y$ ← solve for y

$\dfrac{(x-8)^9+6}{4} = y = f^{-1}(x)$

14. Use algebra to find the inverse of the given one-to-one function. $f(x) = \dfrac{9x}{6x+2}$

Possibilities:

(a) $f^{-1}(x) = \dfrac{2x}{9x+6}$

(b) $f^{-1}(x) = \dfrac{3}{2}x + 2$

(c) $f^{-1}(x) = \dfrac{2x}{9-6x}$

(d) $f^{-1}(x) = \dfrac{9x}{6x-2}$

(e) $f^{-1}(x) = \dfrac{6x+2}{9x}$

$y = \dfrac{9x}{6x+2}$

$x = \dfrac{9y}{6y+2}$

$x(6y+2) = 9y$

$6xy + 2x = 9y$

$6xy - 9y = -2x$

$y(6x-9) = -2x$

$\dfrac{y(6x-9)}{6x-9} = \dfrac{-2x}{6x-9}$

$y = \dfrac{-2x}{-(9-6x)}$

$y = \dfrac{2x}{9-6x}$

$f^{-1}(x) = \dfrac{2x}{9-6x}$

15. Write the given expression without using radicals.

$$\sqrt[13]{x^3} + 17$$

Possibilities:

(a) $x^{13/3} + 17$

(b) $x^{20} - x^{30}$

(c) $x^{3/13} + 17$

(d) x^{27}

(e) x^7

$(x^3)^{1/13} + 17$

$x^{3/13} + 17$

16. A genetic engineer is growing cells in a fermenter. The cells multiply by splitting in half every 15 minutes. The new cells have the same DNA as the original ones. The following table shows the cell population for the first hour.

Time (hours)	Number of Cells
0.00	600
0.25	1200
0.50	2400
0.75	4800
1.00	9600

$*C(t) = P_0 a^t$

Write the rule of the function that gives the number C of cells at time t hours.

Possibilities:

(a) $C(t) = 600\,(2^t)$

(b) $C(t) = 600\,(2^{4t})$

(c) $C(t) = 300\,(2^{4t})$

(d) $C(t) = 15\,(600^t)$

(e) $C(t) = 300\,(2^t)$

$C(0) = P_0 a^0$

$600 = P_0$

$C(t) = 600 a^t$

$C(1) = 600 a^1$

$9600 = 600 a$

$2^4 = 16 = a$

$\longrightarrow C(t) = 600(2^4)^t$

17. A certain fungus grows in a circular shape. Its diameter $D(t)$ in inches after t weeks is given below.

$$D(t) = 9 - \frac{90}{t^2 + 2}$$

How much area is covered by the fungus after 6 weeks?

Possibilities:

(a) about 138.16 square inches
(b) about 28.27 square inches
(c) about 3.32 square inches
(d) about 34.54 square inches
(e) about 6.63 square inches

d = diameter

$* A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$

$A(t) = \pi \left(\frac{D(t)}{2}\right)^2$

$A(6) = \pi \left(\frac{D(6)}{2}\right)^2$

$A(6) = \pi \left(\frac{9 - \frac{90}{6^2+2}}{2}\right)^2$

$A(6) = \pi \left(\frac{9 - \frac{90}{38}}{2}\right)^2 \approx 34.54 \text{ in}^2$

18. Determine how much money will be in a savings account if the initial deposit was \$150 and the interest rate is 3.25% compounded continuously for 2 years, 7 months. (Round your answer to the nearest cent.)

Possibilities:

(a) \$163.14
(b) \$163.36
(c) \$163.58
(d) \$163.80
(e) \$164.02

$* P(t) = P_0 e^{rt} *$

$t = 2\,7/12 \text{ years} \approx 2.5833 \text{ years}$

$P(2.5833) = 150 e^{(.0325)(2.5833)}$

$P(2.5833) = 163.137359\ldots$

$= \$163.14$

19. Translate the given exponential statement into an equivalent logarithmic statement.

$9^x = 6$

base

Possibilities:

(a) $\log_6(9) = x$

(b) $\log_9(6) = x$ → $\log_9(6) = x$

(c) $\log_9(x) = 6$

(d) $\log_x(9) = 6$

(e) $\log_6(x) = 9$

$\log_b(y) = x \iff b^x = y$

"if and only if"

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20. Write the domain of the function $h(x) = \log(x - 17)$ in interval notation.

Possibilities:

(a) $(-\infty, -17)$

(b) $(17, \infty)$

(c) $(-\infty, \infty)$

(d) $(-\infty, 17]$

(e) $(-\infty, 17) \cup (17, \infty)$

$x - 17 > 0$

$x > 17$

17

$(17, \infty)$