MA109 — College Algebra Fall 2015
Final Exam 2015-12-16

Name: KEY Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) (b) (c) (d) (e)	9. (a) (b) (c) (d) (e)	17. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)	10. (a) (b) (c) (d) (e)	18. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)	11. (a) (b) (c) (d) (e)	19. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)	12. (a) (b) (c) (d) (e)	20. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)	13. (a) (b) (c) (d) (e)	21. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)	14. (a) (b) (c) (d) (e)	22. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)	15. (a) (b) (c) (d) (e)	23. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)	16. (a) (b) (c) (d) (e)	

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve for r in:

$$(4r - 36)(r^2 - 25) = 0$$

$4r - 36 = 0$ $\quad$ $r^2 - 25 = 0$

$4r = 36$ $\quad$ $r^2 = 25$

$r = 9$ $\quad$ $r = \pm 5$

Possibilities:

(a) The only real solutions are 9 and ± 5.
(b) The only real solutions are 4 and 25.
(c) The only real solutions are ± 5.
(d) The only real solutions are 36 and 25.
(e) The only real solutions are 4 and 0.

2. For which of the following equations is the number 3 a solution?

Possibilities:

(a) $3x^2 - 2x - 8 = 16$
(b) $|2x| = -6$
(c) $3x^2 - 6 = 0$
(d) $4(6 - x) = 12$
(e) $\frac{4}{x} + 2 = \frac{1}{x - 3}$

(a) $3(3)^2 - 2(3) - 8 \stackrel{?}{=} 16$

$27 - 6 - 8 \stackrel{?}{=} 16$

$13 \neq 16$

(b) $|2(3)| \stackrel{?}{=} -6$

$6 \neq -6$

(c) $3(3)^2 - 6 \stackrel{?}{=} 0$

$21 \neq 0$

(d) $4(6 - 3) \stackrel{?}{=} 12$

$12 \stackrel{\checkmark}{=} 12$

(e) $\frac{4}{3} + 2 \stackrel{?}{=} \frac{1}{3 - 3}$

$\frac{10}{3} \neq$ undefined

3. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -2 \\ x^2 + 3 & \text{if } -2 < x \leq 1 \\ -2x - 5 & \text{if } x > 1 \end{cases}$$

$\Longrightarrow$ $2 = 4$, $4 > 1$

Find $f(4)$.

Possibilities:

(a) -13
(b) 209
(c) 19
(d) 11
(e) 4

$f(4) = -2(4) - 5$

$= -8 - 5$

$= -13$

4. Solve for z.

$$2z^2 - 9z + 3 = 0$$

$a=2 \quad b=-9 \quad c=3$

use quadratic formula

$$z = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(3)}}{2(2)}$$

$$z = \frac{9 \pm \sqrt{81-24}}{4}$$

$$\boxed{z = \frac{9 \pm \sqrt{57}}{4}}$$

Possibilities:

(a) $\dfrac{-9 \pm \sqrt{57}}{4}$

(b) $\dfrac{-9 \pm \sqrt{105}}{4}$

(c) $\dfrac{9 \pm \sqrt{105}}{4}$

(d) $\dfrac{9 \pm \sqrt{57}}{4}$

(e) $\dfrac{9}{4} \pm \sqrt{75}$

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5. Write the given expression as a single logarithm.

$$3\log(x) + \log(4y) - \log(9z)$$

$$\log(x^3) + \log(4y) - \log(9z)$$

$$\log[(x^3)(4y)] - \log(9z)$$

$$\boxed{\log\left[\frac{(x^3)(4y)}{9z}\right]}$$

Possibilities:

(a) $\log\left(\dfrac{x^3(4y)}{9z}\right)$

(b) $\log(3x(4+y) - 9 - z)$

(c) $\log(3x + 4y - 9z)$

(d) $\log\left(\dfrac{x^3y^4}{z^9}\right)$

(e) $\log(x^3y^4z^9)$

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6. Let $f(x) = 4^x$. Which of the following is $f^{-1}(64)$?

$$64 = 4^x$$

$$4^3 = 4^x$$

$$3 = x$$

$$f(3) = 64 \implies f^{-1}(64) = \boxed{3}$$

Possibilities:

(a) $\frac{1}{4}$

(b) 16

(c) 2

(d) 3

(e) $\frac{1}{16}$

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7. The number of bacteria in a culture is modeled by the function $n(t) = 60e^{0.3t}$ where t is measured in hours. When will the number of bacteria reach 2500? Round your answer to the nearest hundredth of an hour.

Possibilities:

(a) About 13.15 hours
(b) About 51.09 hours
(c) About 12.43 hours
(d) About 5.40 hours
(e) About 3.73 hours

$2500 = 60e^{0.3t}$

$\frac{2500}{60} = e^{0.3t}$

$\ln\left(\frac{2500}{60}\right) = \ln\left(e^{0.3t}\right)$

$\ln\left(\frac{2500}{60}\right) = 0.3t$

$\frac{\ln\left(\frac{2500}{60}\right)}{0.3} = t$

$t \approx 12.43$ hrs.

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8. Find an equation for the line through the points $(-4, 7)$ and $(5, 12)$.

Possibilities:

(a) $y - 7 = \frac{5}{9}(x + 4)$
(b) $y - 4 = -\frac{9}{5}(x - 7)$
(c) $y + 7 = \frac{5}{9}(x - 4)$
(d) $y - 5 = \frac{5}{9}(x - 12)$
(e) $y - 12 = -\frac{9}{5}(x - 5)$

$x_1, y_1 \quad x_2, y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 7}{5 - (-4)} = \frac{5}{9}$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{5}{9}[x - (-4)]$

$y - 7 = \frac{5}{9}(x + 4)$

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9. Which of the following statements best describes the system of equations?

$$\begin{cases} x + y = 7 \\ 2x + 2y = 8 \end{cases}$$

$(x + y = 7) \cdot -2 \Rightarrow -2x - 2y = -14$

$2x + 2y = 8$

$+$

$0 \neq -6$

false statement

Possibilities:

(a) The system is dependent. Two solutions to the system are $(4, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$.
(b) The system is inconsistent. Therefore the system has no solutions.
(c) The system is consistent. It has exactly one solution which is $(1, 6)$.
(d) The system is dependent. Every point is a solution to the system.
(e) The system is dependent. Two solutions to the system are $(1, 1)$ and $(7, 8)$. One point that is NOT a solution to the system is $(0, 0)$.

inconsistent system with no solutions

10. A merchant wants to mix peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 39 pounds of a nut mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?

Possibilities:

(a) 4.5 pounds

(b) 20.8 pounds

(c) 32.7 pounds

(d) 113.1 pounds

(e) 15.6 pounds

Value of peanuts + Value of cashews = Value of mixture

x = pounds of peanuts needed

$1.50(x) + 4.50(39-x) = 2.90(39)$

$1.5x + 175.5 - 4.5x = 113.1$

$-3x = -62.4$

$x = 20.8$ pounds

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11. Let $f(x) = 3x^2 - 1$. Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $18x + 9h$

(b) $\dfrac{6xh + 3h^2 - 2}{h}$

(c) 1

(d) $3h$

(e) $6x + 3h$

$f(x) = 3x^2 - 1$

$f(x+h) = 3(x+h)^2 - 1$

$= 3(x^2 + 2xh + h^2) - 1$

$= 3x^2 + 6xh + 3h^2 - 1$

$f(x+h) - f(x) = (3x^2 + 6xh + 3h^2 - 1) - (3x^2 - 1)$

$= 3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h)}{h} = 6x + 3h$

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12. Let $g(x) = \sqrt{x^2 - 4}$. Find the domain of $g(x)$.

Possibilities:

(a) $[2, \infty)$

(b) $(-\infty, -2] \cup [2, \infty)$

(c) $(-\infty, -2) \cup (2, \infty)$

(d) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

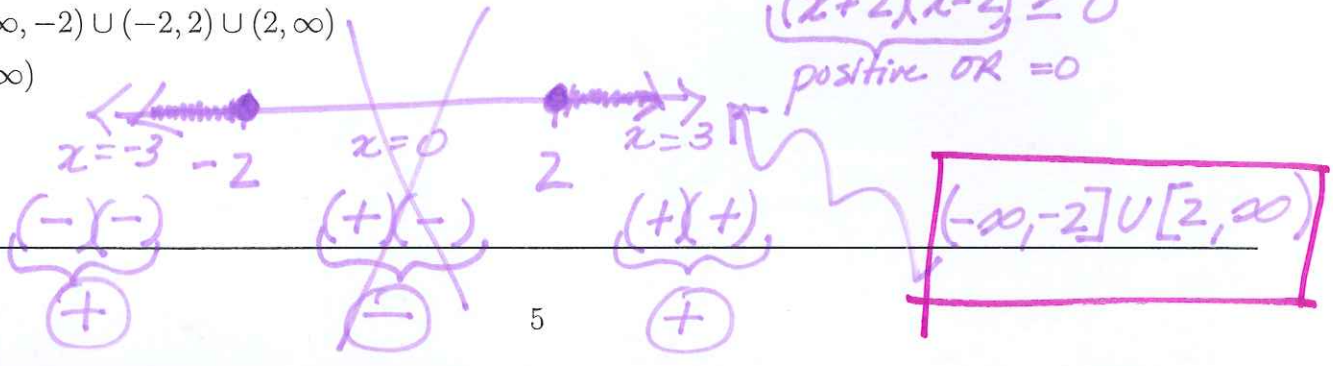
(e) $(2, \infty)$

even root $\Rightarrow$ expression under root ≥ 0

$x^2 - 4 \geq 0$

$(x+2)(x-2) \geq 0$

positive OR $= 0$



$(-\infty, -2] \cup [2, \infty)$

The next three problems refer to the same function.

$$\mathbf{P(x) = x^3 - 11x^2 + 32x - 28}$$

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13. Which of the following is a factor of $P(x)$? (*See the top of the page.*)

Possibilities:

(a) $(x-1)$
(b) $(x-5)$
(c) $(x-4)$
(d) $(x-3)$
(e) $(x-2)$

(a) $P(1) = 1^3 - 11(1)^2 + 32(1) - 28$
$= 1 - 11 + 32 - 28$
$= -6 \leftarrow$ NOT 0

(b) $P(5) = 5^3 - 11(5)^2 + 32(5) - 28$
$= 125 - 275 + 160 - 28$
$= -18 \leftarrow$ NOT 0

(c) $P(4) = 4^3 - 11(4)^2 + 32(4) - 28$
$= -12 \leftarrow$ NOT 0

(d) $P(3) = 3^3 - 11(3)^2 + 32(3) - 28$
$= -4 \leftarrow$ NOT 0

(e) $P(2) = 2^3 - 11(2)^2 + 32(2) - 28$
$= 8 - 11(4) + 64 - 28$
$= 8 - 44 + 64 - 28$
$= 72 - 72$
$= 0$

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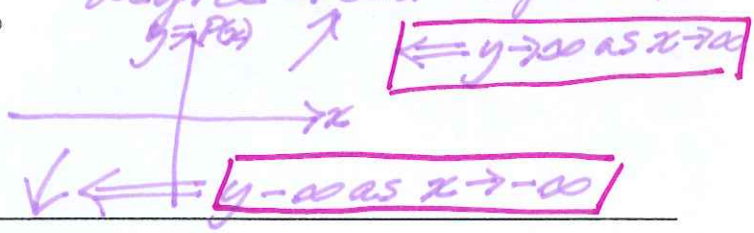
14. Determine the end behavior of the graph of $y = P(x)$. (*See the top of the page.*)

Possibilities:

(a) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(b) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(c) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(d) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(e) No solution

End behavior determined by degree and sign of leading term, x^3
degree $\Rightarrow$ odd sign $\Rightarrow +$



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15. Find the remainder of the division problem $\dfrac{P(x)}{x+3}$. (*See the top of the page.*)

Possibilities:

(a) 194
(b) $74x - 28$
(c) $x^2 - 1$
(d) -28
(e) -250

$P(-3) =$ remainder

$P(-3) = (-3)^3 - 11(-3)^2 + 32(-3) - 28$
$= -27 - 11(9) - 96 - 28$
$= -27 - 99 - 96 - 28$
$= -250$

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16. Suppose the graph of $y = f(x)$ is a parabola with vertex $(-1, 2)$ and goes through the points $(0, 6)$. Which of the following is an formula for $f(x)$?

Possibilities:

(a) $f(x) = 4(x+2)^2 - 1$

(b) $f(x) = 4(x+1)^2 + 2$

(c) $f(x) = (x-1)^2 + 2$

(d) $f(x) = (x+2)(x+3)$

(e) $f(x) = (x+1)(x+6)$

h k x y

standard "vertex" form $\Rightarrow y = a(x-h)^2 + k$

$6 = a(0+1)^2 + 2$

$4 = a$

$y = 4(x+1)^2 + 2$

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17. Solve for x.

$$6\log_4(x+5) = 12$$

Possibilities:

(a) $x = \sqrt[6]{12}$

(b) $x = \dfrac{12}{6\log(4)}$

(c) $x = 11$

(d) $x = -4.5$

(e) $x = 0$

$\log_4(x+5) = \frac{12}{6}$

rewrite in exponential form

$4^2 = x+5$

$16 - 5 = x$

$11 = x$

---

18. Let $P(x) = 7x^{50} + 4x^{40} - 31x^{30} + 3x^{20} + 4$. List all possible rational zeros of $P(x)$ given by the Rational Zeros Theorem (but do not check to see which are actually zeros).

Possibilities:

(a) $\pm 1, \pm 4, \pm 7/4$

(b) $\pm 1, \pm 1/2, \pm 1/4, \pm 7, \pm 7/2, \pm 7/4$

(c) $\pm 1, \pm 2, \pm 4, \pm 1/7, \pm 2/7, \pm 4/7$

(d) $\pm 1, \pm 2, \pm 4, \pm 7, \pm 7/2, \pm 7/4$

(e) $\pm 1, \pm 4, \pm 4/7$

$a_0 = 4 \Rightarrow$ factors: $\pm 1, \pm 2, \pm 4$

$a_n = 7 \Rightarrow$ factors: $\pm 1, \pm 7$

possible rational roots:

$\frac{\text{factors of } a_0}{\text{factors of } a_n} \Rightarrow \pm\frac{1}{1}, \pm\frac{1}{7}, \pm\frac{2}{1}, \pm\frac{2}{7}, \pm\frac{4}{1}, \pm\frac{4}{7}$

$\pm 1, \pm\frac{1}{7}, \pm 2, \pm\frac{2}{7}, \pm 4, \pm\frac{4}{7}$

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19. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = 2160p - 120p^2.$$

What is the team's maximum revenue?

* max/min of quadratic occur at vertex

Possibilities:

(a) \$10360

(b) \$9

(c) \$8

(d) \$9720

(e) \$9980

Vertex $\Rightarrow (h, k) = \left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right)\right)$

$h = \frac{-2160}{2(-120)} = 9$ ← price to achieve max Revenue

$k = R(9) = 2160(9) - 120(9)^2$ ← max Revenue

$= 19440 - 9720$

$= \$9720$

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20. Let $r(x) = \dfrac{x+4}{x+7}$. Find the asymptotes of r.

V.A. $\Rightarrow$ occur at zeros of denominator that are NOT 0s of numerator

$x+7=0 \Rightarrow$ $x=-7$ is V.A.

Possibilities:

(a) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = -4$.

(b) The vertical asymptote is $x = 1$ and the horizontal asymptote is $y = -7$.

(c) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = -7$.

(d) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = 1$.

(e) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = 1$.

H.A. $\Rightarrow$ occur at ratio of leading terms of numerator & denominator

$\frac{x}{x} = 1 \Rightarrow$ $y=1$ is H.A.

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21. Explain how the graph of $g(x) = (x+5)^2 - 8$ is obtained from the graph of $f(x) = x^2$.

shift left 5

shift down 8

Possibilities:

(a) Shift the graph of f right 5 units and shift up 8 units to obtain the graph of g.

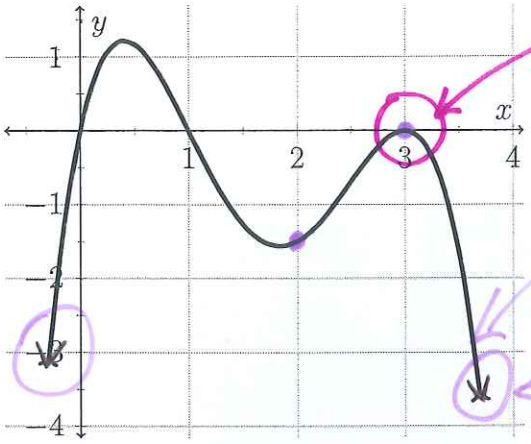
(b) Shift the graph of f left 5 units and shift down 8 units to obtain the graph of g.

(c) Shift the graph of f right 5 units and shift down 8 units to obtain the graph of g.

(d) Shift the graph of f right 8 units and shift up 5 units to obtain the graph of g.

(e) Shift the graph of f left 8 units and shift down 5 units to obtain the graph of g.

The next two problems refer to the graph shown. In the picture below, the graph of the polynomial function $P(x)$ is shown.



root touches, but does not cross x-axis

both ends going down indicates negative sign of leading coefficient

both ends going in same direction indicates even degree

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22. For the graph of the polynomial $P(x)$ drawn above, which of the following can you conclude about P?

since both ends go in same direction → even

end behavior is determined by degree & sign of leading coefficient

Possibilities:

(a) The degree of the polynomial is odd and the leading coefficient is negative.

(b) The parity (even or odd) of the degree of the polynomial or the sign of the leading coefficient can not be determined by the graph.

even & negative ← because pointing down

(c) The degree of the polynomial is even and the leading coefficient is positive.

(d) The degree of the polynomial is odd and the leading coefficient is positive.

(e) The degree of the polynomial is even and the leading coefficient is negative.

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23. For the graph of the polynomial $P(x)$ drawn above, which of the following statements can be concluded?

(I). $(x+1)$ is a factor of $P(x)$ $\Longrightarrow$ -1 is NOT an x-intercept

(II). When $P(x)$ is divided by $(x-2)$ the remainder is six. $P(2) \approx -1.5$ $P(2) \neq 6$

(III). $x = 3$ is a root with even multiplicity.

roots with even multiplicity touch (but do NOT cross) x-axis

Possibilities:

(a) Only statements (I) and (II) are true.

(b) None of the statements are true.

(c) Only statement (III) is true.

(d) Only statement (II) is true.

(e) Statements (I), (II), and (III) are all true.

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$