MA109 — College Algebra	Fall 2015	Name: KEY	Sec.: ______
Final Exam	2015-12-16		

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) (b) (c) (d) (e)	9. (a) (b) (c) (d) (e)	17. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)	10. (a) (b) (c) (d) (e)	18. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)	11. (a) (b) (c) (d) (e)	19. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)	12. (a) (b) (c) (d) (e)	20. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)	13. (a) (b) (c) (d) (e)	21. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)	14. (a) (b) (c) (d) (e)	22. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)	15. (a) (b) (c) (d) (e)	23. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)	16. (a) (b) (c) (d) (e)	

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: ______________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve for r in:

$$(4r - 36)(r^2 - 25) = 0$$

By the Zero Product property,
$(4r-36)(r^2-25)=0$ implies either

$4r-36=0$ or $r^2-25=0$
$4r=36$ $\quad$ $r^2=25$
$r=9$ $\quad$ $r=\pm\sqrt{25}$
$r=\pm 5$

Possibilities:

(a) The only real solutions are 9 and ± 5.
(b) The only real solutions are 4 and 25.
(c) The only real solutions are ± 5.
(d) The only real solutions are 36 and 25.
(e) The only real solutions are 4 and 0.

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2. For which of the following equations is the number 3 a solution?

3 is a solution if it makes the equation true when you plug it in for x.

Checking we see $4(6-3)=4(3)=12$ ✓.

Possibilities:

(a) $3x^2 - 2x - 8 = 16$
(b) $|2x| = -6$
(c) $3x^2 - 6 = 0$
(d) $4(6 - x) = 12$
(e) $\frac{4}{x} + 2 = \frac{1}{x-3}$

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3. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \le -2 \\ x^2 + 3 & \text{if } -2 < x \le 1 \\ -2x - 5 & \text{if } x > 1 \end{cases}$$

Find $f(4)$.

Since $4 > 1$, $f(4) = -2(4) - 5$
$= -8 - 5 = -13$

Possibilities:

(a) -13
(b) 209
(c) 19
(d) 11
(e) 4

4. Solve for z.

$$2z^2 - 9z + 3 = 0$$

Using the quadratic formula, we got

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81-24}}{4} = \frac{9 \pm \sqrt{57}}{4}$$

Possibilities:

(a) $\dfrac{-9 \pm \sqrt{57}}{4}$

(b) $\dfrac{-9 \pm \sqrt{105}}{4}$

(c) $\dfrac{9 \pm \sqrt{105}}{4}$

(d) $\dfrac{9 \pm \sqrt{57}}{4}$

(e) $\dfrac{9}{4} \pm \sqrt{75}$

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5. Write the given expression as a single logarithm.

$$3\log(x) + \log(4y) - \log(9z)$$

$$= \log(x^3) + \log(4y) - \log(9z)$$

$$= \log(x^3(4y)) - \log(9z)$$

$$= \log\left(\frac{x^3(4y)}{9z}\right)$$

Possibilities:

(a) $\log\left(\dfrac{x^3(4y)}{9z}\right)$

(b) $\log\left(3x(4+y) - 9 - z\right)$

(c) $\log\left(3x + 4y - 9z\right)$

(d) $\log\left(\dfrac{x^3y^4}{z^9}\right)$

(e) $\log\left(x^3y^4z^9\right)$

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6. Let $f(x) = 4^x$. Which of the following is $f^{-1}(64)$?

Possibilities:

(a) $\frac{1}{4}$

(b) 16

(c) 2

(d) 3

(e) $\frac{1}{16}$

To find $f^{-1}(64)$, we need to find x such that $f(x) = 64$. Then $x = f^{-1}(f(x)) = f^{-1}(64)$

For $f(x) = 4^x = 64$, we solve by taking $\log_4$ of both sides: $\log_4(4^x) = \log_4(64)$

$$x = 3$$

7. The number of bacteria in a culture is modeled by the function $n(t) = 60e^{0.3t}$ where t is measured in hours. When will the number of bacteria reach 2500? Round your answer to the nearest hundredth of an hour.

Possibilities:

(a) About 13.15 hours

(b) About 51.09 hours

(c) About 12.43 hours

(d) About 5.40 hours

(e) About 3.73 hours

$$2500 = 60e^{0.3t}$$

$$\frac{2500}{60} = e^{0.3t}$$

$$\ln\left(\frac{2500}{60}\right) = \ln\left(e^{0.3t}\right)$$

$$\ln\left(\frac{2500}{60}\right) = 0.3t$$

$$\frac{\ln\left(\frac{2500}{60}\right)}{0.3} = t \qquad \frac{\ln\left(\frac{2500}{60}\right)}{0.3} \approx 12.43$$

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8. Find an equation for the line through the points $(-4, 7)$ and $(5, 12)$.

Possibilities:

(a) $y - 7 = \frac{5}{9}(x + 4)$

(b) $y - 4 = -\frac{9}{5}(x - 7)$

(c) $y + 7 = \frac{5}{9}(x - 4)$

(d) $y - 5 = \frac{5}{9}(x - 12)$

(e) $y - 12 = -\frac{9}{5}(x - 5)$

Point Slope form of a line is $y - y_1 = m(x - x_1)$

To find m, we use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

so $m = \frac{12-7}{5-(-4)} = \frac{5}{9}$. Since we know two points, we are either going to get

$y - 12 = \frac{5}{9}(x-5)$ or $y - 7 = \frac{5}{9}(x+4)$

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9. Which of the following statements best describes the system of equations?

$$\begin{cases} x + y = 7 \\ 2x + 2y = 8 \end{cases}$$

Multiplying the first equation by 2, and subtracting the second from the first we get

$$\begin{array}{r} 2x + 2y = 14 \\ -(2x + 2y = 8) \\ \hline 0 = 6 \end{array}$$

Since $0 \neq 6$, the system is inconsistent and therefore has no solutions.

Possibilities:

(a) The system is dependent. Two solutions to the system are $(4, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$.

(b) The system is inconsistent. Therefore the system has no solutions.

(c) The system is consistent. It has exactly one solution which is $(1, 6)$.

(d) The system is dependent. Every point is a solution to the system.

(e) The system is dependent. Two solutions to the system are $(1, 1)$ and $(7, 8)$. One point that is NOT a solution to the system is $(0, 0)$.

10. A merchant wants to mix peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 39 pounds of a nut mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?

Let x = # of pounds of peanuts
y = # of pounds of cashews.

Possibilities:

(a) 4.5 pounds

(b) 20.8 pounds

(c) 32.7 pounds

(d) 113.1 pounds

(e) 15.6 pounds

Then $x+y=39$.

Since we want the mixture to cost \$2.90 per pound, its total cost is \$$39\cdot 2.90 =$ \$113.10. Thus

$1.50x + 4.50y = 113.10$

Solving for y, we get $y = 39-x$

So $1.50x + 4.50(39-x) = 113.10$

$1.50x + 175.5 - 4.50x = 113.10$

$62.4 = 3x$

$x = 20.8$

11. Let $f(x) = 3x^2 - 1$. Find $\dfrac{f(x+h)-f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $18x + 9h$

(b) $\dfrac{6xh + 3h^2 - 2}{h}$

(c) 1

(d) $3h$

(e) $6x + 3h$

$$\frac{f(x+h)-f(x)}{h} = \frac{(3(x+h)^2-1)-(3x^2-1)}{h}$$

$$= \frac{(3(x^2+2xh+h^2)-1)-3x^2+1}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2\cancel{-1}\cancel{-3x^2}\cancel{+1}}{h}$$

$$= \frac{6xh+3h^2}{h} = 6x+3h$$

12. Let $g(x) = \sqrt{x^2-4}$. Find the domain of $g(x)$.

Possibilities:

(a) $[2,\infty)$

(b) $(-\infty,-2] \cup [2,\infty)$

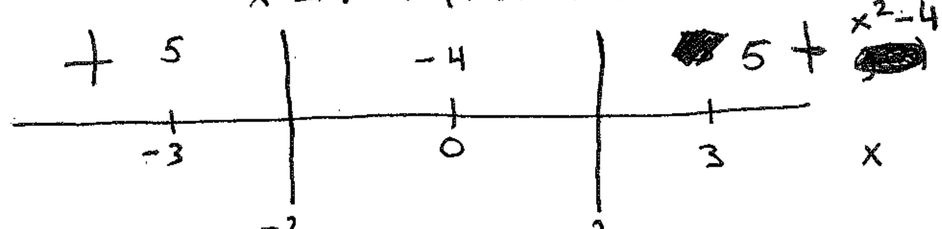
(c) $(-\infty,-2) \cup (2,\infty)$

(d) $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

(e) $(2,\infty)$

For x to be in the domain of $g(x) = \sqrt{x^2-4}$, we need $x^2-4 \geq 0$. To solve this, we find the solutions of $x^2=4$ and create a sign chart.

$x^2=4$ implies $x = \pm 2$



Thus $x^2-4 \geq 0$ in the domain $(-\infty,-2] \cup [2,\infty)$

The next three problems refer to the same function.

$$\boxed{P(x) = x^3 - 11x^2 + 32x - 28}$$

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13. Which of the following is a factor of $P(x)$? *(See the top of the page.)*

Possibilities:

(a) $(x-1)$
(b) $(x-5)$
(c) $(x-4)$
(d) $(x-3)$
(e) $(x-2)$

Recall that $(x-c)$ is a factor of $P(x)$ if and only if c is a root of P. One could simply calculate $P(1)$, $P(2)$, $P(3)$, $P(4)$, and $P(5)$. Alternatively, we could use the Rational Roots Theorem to see that if $\frac{r}{s}$ is a rational root, then r divides 28 and s divides 1. Thus $\frac{r}{s} = \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$. Thus, we need only check (a), (c), or (e). Then $P(1) = -6$, $P(4) = -12$ and $P(2) = 0$. Thus $(x-2)$ is a factor of $P(x)$.
(If you are using a graphing calculator, you can simply find the x-intercept of $P(x)$.)

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14. Determine the end behavior of the graph of $y = P(x)$. *(See the top of the page.)*

Possibilities:

(a) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(b) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(c) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(d) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(e) No solution

When determining the end behavior, we need only consider the leading term, in this case x^3.

Since the leading coefficient is positive and the degree is odd,
$y \to \infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

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15. Find the remainder of the division problem $\frac{P(x)}{x+3}$. *(See the top of the page.)*

Possibilities:

(a) 194
(b) $74x - 28$
(c) $x^2 - 1$
(d) -28
(e) -250

By the Remainder Theorem, the remainder of $\frac{P(x)}{x+3}$ is equal to $P(-3)$.

$$P(-3) = (-3)^3 - 11(-3)^2 + 32(-3) - 28$$
$$= -250$$

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16. Suppose the graph of $y = f(x)$ is a parabola with vertex $(-1, 2)$ and goes through the points $(0, 6)$. Which of the following is an formula for $f(x)$?

Possibilities:

(a) $f(x) = 4(x+2)^2 - 1$

(b) $f(x) = 4(x+1)^2 + 2$

(c) $f(x) = (x-1)^2 + 2$

(d) $f(x) = (x+2)(x+3)$

(e) $f(x) = (x+1)(x+6)$

The standard form of a quadratic function whose parabola has vertex (h,k) is $f(x) = a(x-h)^2 + k$.

Thus $f(x) = a(x+1)^2 + 2$. To find a, note $f(0) = 6$ since the graph of f goes through the point $(0,6)$. Thus

$$6 = f(0) = a(0+1)^2 + 2 = a + 2$$

$$4 = a \quad \text{so} \quad f(x) = 4(x+1)^2 + 2$$

17. Solve for x.

$$6 \log_4 (x+5) = 12$$

Possibilities:

(a) $x = \sqrt[6]{12}$

(b) $x = \dfrac{12}{6\log(4)}$

(c) $x = 11$

(d) $x = -4.5$

(e) $x = 0$

$$\log_4 (x+5) = 2$$

$$\cancel{4}^{\cancel{\log_4}(x+5)} = 4^2$$

$$x + 5 = 4^2$$

$$x + 5 = 16$$

$$x = 11$$

18. Let $P(x) = 7x^{50} + 4x^{40} - 31x^{30} + 3x^{20} + 4$. List all possible rational zeros of $P(x)$ given by the Rational Zeros Theorem (but do not check to see which are actually zeros).

Possibilities:

(a) $\pm 1, \pm 4, \pm 7/4$

(b) $\pm 1, \pm 1/2, \pm 1/4, \pm 7, \pm 7/2, \pm 7/4$

(c) $\pm 1, \pm 2, \pm 4, \pm 1/7, \pm 2/7, \pm 4/7$

(d) $\pm 1, \pm 2, \pm 4, \pm 7, \pm 7/2, \pm 7/4$

(e) $\pm 1, \pm 4, \pm 4/7$

If $\frac{r}{s}$ is a rational root in lowest terms, then r divides 4 and s divides 7.

Thus $r = \pm 1, \pm 2, \pm 4$ and $s = \pm 1, \pm 7$

Thus $\frac{r}{s} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{7}, \pm \frac{2}{7}, \pm \frac{4}{7}$

[redacted]

19. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = 2160p - 120p^2.$$

What is the team's maximum revenue?

Note that $R(p)$ is a quadratic function with a negative leading term. Therefore it has its maximum value at the vertex, Since we know that the vertex of $y = ax^2 + bx + c$ is always at $x = \frac{-b}{2a}$, we see that $R(p)$ has its vertex at $p = \frac{-2160}{-240} = 9$ and its maximum value is

$R(9) = 2160(9) - 120(9^2)$
$= 9720$

Possibilities:

(a) \$10360

(b) \$9

(c) \$8

(d) \$9720

(e) \$9980

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20. Let $r(x) = \dfrac{x+4}{x+7}$. Find the asymptotes of r.

Horizontal asymptote comes from ratio of leading terms $\frac{x}{x} = 1$, so $y = 1$ is the horizontal asymptote. Vertical asymptotes are at x values that are roots of the denominator but not the numerator.

Since $(-7)+7=0$ and $(-7)+4 \neq 0$, we see that $x=-7$ is a vertical asymptote.

Possibilities:

(a) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = -4$.

(b) The vertical asymptote is $x = 1$ and the horizontal asymptote is $y = -7$.

(c) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = -7$.

(d) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = 1$.

(e) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = 1$.

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21. Explain how the graph of $g(x) = (x+5)^2 - 8$ is obtained from the graph of $f(x) = x^2$.

$g(x) = f(x+5) - 8$ so it is obtained by shifting left 5 and down 8.

Possibilities:

(a) Shift the graph of f right 5 units and shift up 8 units to obtain the graph of g.

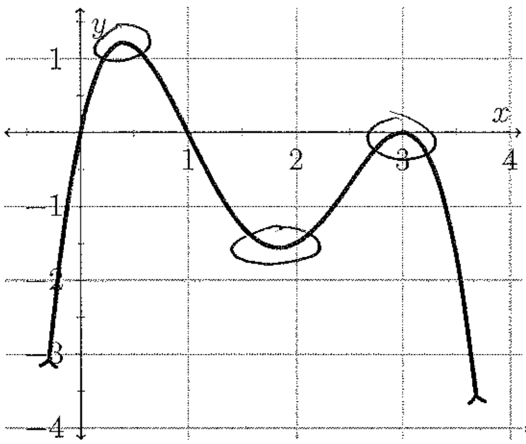
(b) Shift the graph of f left 5 units and shift down 8 units to obtain the graph of g.

(c) Shift the graph of f right 5 units and shift down 8 units to obtain the graph of g.

(d) Shift the graph of f right 8 units and shift up 5 units to obtain the graph of g.

(e) Shift the graph of f left 8 units and shift down 5 units to obtain the graph of g.

The next two problems refer to the graph shown. In the picture below, the graph of the polynomial function $P(x)$ is shown.



Recall that the number of extrema of a graph of a polynomial $P(x)$ of degree n is at most $n-1$. Furthermore, the graph of $P(x)$ has an odd number of extrema if the degree of $P(x)$ is even, and an even number of extrema if the degree of $P(x)$ is odd. Finally, the end behavior of the graph tells us if the leading coefficient is positive or negative.

22. For the graph of the polynomial $P(x)$ drawn above, which of the following can you conclude about P?

Since the graph of $P(x)$ has 3 extrema, the degree of P must be at least 4 and it must be even. Since $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$, the leading coefficient is negative.

Possibilities:

(a) The degree of the polynomial is odd and the leading coefficient is negative.

(b) The parity (even or odd) of the degree of the polynomial or the sign of the leading coefficient can not be determined by the graph.

(c) The degree of the polynomial is even and the leading coefficient is positive.

(d) The degree of the polynomial is odd and the leading coefficient is positive.

(e) The degree of the polynomial is even and the leading coefficient is negative.

23. For the graph of the polynomial $P(x)$ drawn above, which of the following statements can be concluded?

(I). $(x+1)$ is a factor of $P(x)$

(II). When $P(x)$ is divided by $(x-2)$ the remainder is six.

(III). $x = 3$ is a root with even multiplicity.

i) would imply that $P(-1) = 0$ which is false

ii) would imply that $P(2) = 6$ (by the Remainder Theorem) which is false.

iii) is true since $P(3) = 0$ meaning $x = 3$ is a root and the fact that the graph touches the x-axis but doesn't cross it implies that $x = 3$ is a root of even multiplicity.

Possibilities:

(a) Only statements (I) and (II) are true.

(b) None of the statements are true.

(c) Only statement (III) is true.

(d) Only statement (II) is true.

(e) Statements (I), (II), and (III) are all true.

$\boxed{\text{Formula Sheet:}}$

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$