MA109 — College Algebra Final Exam	Fall 2015 2015-12-16	**Name:** detailed KEY **Sec.:** ________

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. **(a)** (b) (c) (d) (e)
2. (a) (b) (c) **(d)** (e)
3. **(a)** (b) (c) (d) (e)
4. (a) (b) (c) **(d)** (e)
5. **(a)** (b) (c) (d) (e)
6. (a) (b) (c) **(d)** (e)
7. (a) (b) **(c)** (d) (e)
8. **(a)** (b) (c) (d) (e)
9. (a) **(b)** (c) (d) (e)
10. (a) **(b)** (c) (d) (e)
11. (a) (b) (c) (d) **(e)**
12. (a) **(b)** (c) (d) (e)
13. (a) (b) (c) (d) **(e)**
14. (a) (b) **(c)** (d) (e)
15. (a) (b) (c) (d) **(e)**
16. (a) **(b)** (c) (d) (e)
17. (a) (b) **(c)** (d) (e)
18. (a) (b) **(c)** (d) (e)
19. (a) (b) (c) **(d)** (e)
20. (a) (b) (c) (d) **(e)**
21. (a) **(b)** (c) (d) (e)
22. (a) (b) (c) (d) **(e)**
23. (a) (b) **(c)** (d) (e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: ____________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Solve for r in:

* factored equation Should use ZERO PRODUCT PROPERTY to solve.

$$(4r - 36)(r^2 - 25) = 0$$

set each factor equal to 0 and solve.

Possibilities:

(a) The only real solutions are 9 and ± 5.
(b) The only real solutions are 4 and 25.
(c) The only real solutions are ± 5.
(d) The only real solutions are 36 and 25.
(e) The only real solutions are 4 and 0.

$4r - 36 = 0$
$+36 \quad +36$
$\frac{4r}{4} = \frac{36}{4}$
$\boxed{r = 9}$

$r^2 - 25 = 0$
$r^2 = 25$
$\sqrt{r^2} = \pm\sqrt{25}$
$\boxed{r = \pm 5}$

2. For which of the following equations is the number 3 a solution?

* test solution by plugging into equations

Possibilities:

(a) $3x^2 - 2x - 8 = 16$
(b) $|2x| = -6$
(c) $3x^2 - 6 = 0$
(d) $4(6 - x) = 12$
(e) $\dfrac{4}{x} + 2 = \dfrac{1}{x-3}$

(a) $3(3)^2 - 2(3) - 8 \stackrel{?}{=} 16$
$27 - 6 - 8 \stackrel{?}{=} 16$
$13 \neq 16$

(b) $|2(3)| \stackrel{?}{=} -6$
$|6| \stackrel{?}{=} -6$
$6 \neq -6$

(c) $3(3)^2 - 6 \stackrel{?}{=} 0$
$27 - 6 \neq 0$

(d) $4(6-3) \stackrel{?}{=} 12$
$4(3) \stackrel{?}{=} 12$
$\boxed{12 \stackrel{\checkmark}{=} 12}$

(e) $\frac{4}{3} + 2 \stackrel{?}{=} \frac{1}{3-3}$
$\frac{10}{3} \neq$ undefined

3. Let

$$f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -2 \\ x^2 + 3 & \text{if } -2 < x \leq 1 \\ \boxed{-2x - 5} & \text{if } x > 1 \end{cases}$$

$x = 4$
$4 > 1$

Find $f(4)$.

* must first determine which rule to use based ~~on~~ INPUT value

Possibilities:

(a) -13
(b) 209
(c) 19
(d) 11
(e) 4

$f(4) = -2(4) - 5$
$= -8 - 5$
$\boxed{= -13}$

4. Solve for z. * Cannot factor because "a"·"c" = (2)(3) = 6 has no 2 factors whose sum is "b" = −9.

$$2z^2 - 9z + 3 = 0$$

easier since $a \neq 1$

* May use quadratic formula OR completing the square to solve.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ MEMORIZE!!!

Possibilities:

(a) $\dfrac{-9 \pm \sqrt{57}}{4}$

(b) $\dfrac{-9 \pm \sqrt{105}}{4}$

(c) $\dfrac{9 \pm \sqrt{105}}{4}$

(d) $\dfrac{9 \pm \sqrt{57}}{4}$

(e) $\dfrac{9}{4} \pm \sqrt{75}$

$a = 2 \quad b = -9 \quad c = 3$

$z = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(3)}}{2(2)}$

$z = \dfrac{9 \pm \sqrt{81 - 24}}{4}$

$\boxed{z = \dfrac{9 \pm \sqrt{57}}{4}}$

5. Write the given expression as a single logarithm.

$$3\log(x) + \log(4y) - \log(9z)$$

* Power Property $n\log_a(x) = \log_a(x)^n$

* Product Property $\log_a(x) + \log_a(y) = \log_a(xy)$

Possibilities:

(a) $\log\left(\dfrac{x^3(4y)}{9z}\right)$

(b) $\log(3x(4+y) - 9 - z)$

(c) $\log(3x + 4y - 9z)$

(d) $\log\left(\dfrac{x^3y^4}{z^9}\right)$

(e) $\log(x^3y^4z^9)$

$\underbrace{\log(x)^3 + \log(4y)} - \log(9z)$

$\underbrace{\log(x^3 \cdot 4y) - \log(9z)}$

* Quotient Property $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$

$\boxed{\log\left(\dfrac{x^3(4y)}{9z}\right)}$

6. Let $f(x) = 4^x$. Which of the following is $f^{-1}(64)$?

* INVERSE functions $f^{-1}(x)$ take the "output" of the function as "input"

inverse input is original output

Possibilities:

(a) $\frac{1}{4}$

(b) 16

(c) 2

(d) 3

(e) $\frac{1}{16}$

$64 = 4^x$

$4^3 = 4^x$

$\boxed{3 = x}$

$f(3) = 4^3$

$f(3) = 64 \Longrightarrow f^{-1}(64) = 3$

7. The number of bacteria in a culture is modeled by the function $n(t) = 60e^{0.3t}$ where t is measured in hours. When will the number of bacteria reach 2500? Round your answer to the nearest hundredth of an hour. *Set function equal to 2500 and solve

Possibilities:

(a) About 13.15 hours

(b) About 51.09 hours

(c) About 12.43 hours

(d) About 5.40 hours

(e) About 3.73 hours

isolate exp. term

ln both sides

use Power Property

$$2500 = 60e^{0.3t}$$

$$\frac{2500}{60} = e^{0.3t}$$

$$\ln\left(\frac{2500}{60}\right) = \ln\left(e^{0.3t}\right)$$

$$\ln\left(\frac{2500}{60}\right) = 0.3t\underbrace{\ln(e)}_{=1}$$

$$\frac{\ln\left(\frac{2500}{60}\right)}{.03} = t$$

$\boxed{12.43 \text{ hrs} \approx t}$

8. Find an equation for the line through the points $(\overset{x_1}{-4}, \overset{y_1}{7})$ and $(\overset{x_2}{5}, \overset{y_2}{12})$.

Possibilities: *equation of a line requires SLOPE!

(a) $y - 7 = \frac{5}{9}(x + 4)$

(b) $y - 4 = -\frac{9}{5}(x - 7)$

(c) $y + 7 = \frac{5}{9}(x - 4)$

(d) $y - 5 = \frac{5}{9}(x - 12)$

(e) $y - 12 = -\frac{9}{5}(x - 5)$

$$m \Rightarrow \frac{\Delta y}{\Delta x} \Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-7}{5-(-4)} = \frac{5}{9}$$

*with a point & slope use PT. SLOPE FORM

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{5}{9}(x - (-4))$$

$$\boxed{y - 7 = 5/9(x + 4)}$$

9. Which of the following statements best describes the system of equations?

*Slope-intercept form will reveal whether lines intersect, are parallel, or coincide.

$$\begin{cases} x + y = 7 \\ 2x + 2y = 8 \end{cases}$$

$$y = -x + 7$$

$$\frac{2y}{2} = \frac{-2x + 8}{2}$$

$$y = -x + 4$$

Possibilities:

(a) The system is dependent. Two solutions to the system are $(4, 3)$ and $(2, 2)$. One point that is NOT a solution to the system is $(1, 1)$.

(b) The system is inconsistent. Therefore the system has no solutions.

(c) The system is consistent. It has exactly one solution which is $(1, 6)$.

(d) The system is dependent. Every point is a solution to the system.

(e) The system is dependent. Two solutions to the system are $(1, 1)$ and $(7, 8)$. One point that is NOT a solution to the system is $(0, 0)$.

same slopes & different y-int. indicate parallel lines

Parallel lines do NOT intersect and thus have no solution

10. A merchant wants to mix peanuts that cost \$1.50 per pound and cashews that cost \$4.50 per pound to obtain 39 pounds of a nut mixture that costs \$2.90 per pound. How many pounds of peanuts are needed?

Possibilities:

(a) 4.5 pounds
(b) 20.8 pounds
(c) 32.7 pounds
(d) 113.1 pounds
(e) 15.6 pounds

*mixture problems can benefit from a system of equations

quantity: $(p + c = 39) \times -4.50$

value: $1.50p + 4.50c = 2.90(39)$

Let p = amount of peanuts & c = amount of cashews

multiply 1st by -4.5

$-4.50p - 4.50c = -4.50(39)$

$-3p = -1.6(39)$

$p = \frac{-1.6(39)}{-3}$

$\boxed{p = 20.8}$

*eliminate "c" since problem asks for "p"

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11. Let $f(x) = 3x^2 - 1$. Find $\frac{f(x+h) - f(x)}{h}$ and simplify. (Assume $h \neq 0$.)

Possibilities:

(a) $18x + 9h$
(b) $\frac{6xh + 3h^2 - 2}{h}$
(c) 1
(d) $3h$
(e) $6x + 3h$

*Need to find $f(x+h)$

plug "x+h" in for "x"

$f(x+h) = 3(x+h)^2 - 1$
$= 3(x^2 + 2xh + h^2) - 1$
$= 3x^2 + 6xh + 3h^2 - 1$

*And then subtract $f(x)$

$f(x+h) - f(x) = (\cancel{3x^2} + 6xh + 3h^2 \cancel{-1}) - (\cancel{3x^2} \cancel{-1})$

*And then divide by h

$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2}{h} = \frac{\cancel{h}(6x + 3h)}{\cancel{h}} = \boxed{6x + 3h}$

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12. Let $g(x) = \sqrt{x^2 - 4}$. Find the domain of $g(x)$.

Possibilities:

(a) $[2, \infty)$
(b) $(-\infty, -2] \cup [2, \infty)$
(c) $(-\infty, -2) \cup (2, \infty)$
(d) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
(e) $(2, \infty)$

*even roots require non-negative values under the root

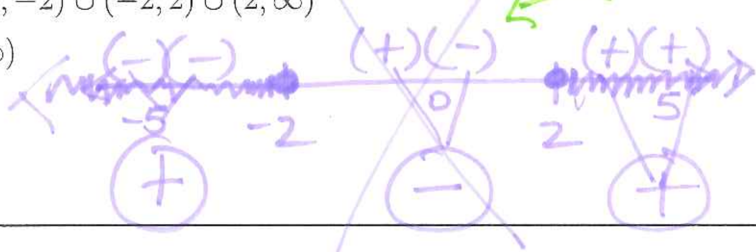
set expression ≥ 0

$x^2 - 4 \geq 0$

$(x+2)(x-2) \geq 0$

solve non-linear inequality

Positive regions & = to 0



$\boxed{(-\infty, -2] \cup [2, \infty)}$

The next three problems refer to the same function.

$$\boxed{\mathrm{P(x) = x^3 - 11x^2 + 32x - 28}}$$

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13. Which of the following is a factor of $P(x)$? (*See the top of the page.*)

Possibilities: *If $P(c)=0$, then $(x-c)$ is a factor. evaluate each in polynomial

(a) $(x-1)$ $P(1)=1^3-11(1)^2+32(1)-28=1-11+32-28=-6\neq 0$

(b) $(x-5)$ $P(5)=5^3-11(5)^2+32(5)-28=125-275+160-28=-18\neq 0$

(c) $(x-4)$ $P(4)=4^3-11(4)^2+32(4)-28=64-176+128-28=-12\neq 0$

(d) $(x-3)$ $P(3)=3^3-11(3)^2+32(3)-28=27-99+96-28=-4\neq 0$

(e) $(x-2)$

$P(2)=2^3-11(2)^2+32(2)-28=8-44+64-28=\boxed{0}$

Since $P(2)=0$, then $(x-2)$ must be a factor.

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14. Determine the end behavior of the graph of $y = P(x)$. (*See the top of the page.*)

Possibilities: *end behavior of a polynomial determined by leading term

(a) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

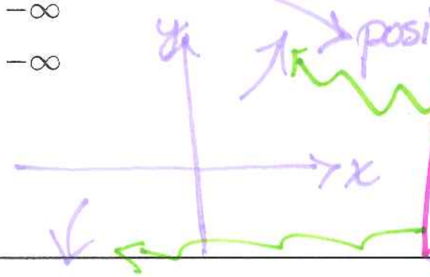
(b) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(c) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(d) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(e) No solution

x^3 → odd degree → positive coefficient



$\boxed{\begin{array}{l} y\to\infty \text{ as } x\to\infty \\ y\to-\infty \text{ as } x\to-\infty \end{array}}$

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15. Find the remainder of the division problem $\dfrac{P(x)}{x+3}$. (*See the top of the page.*) $\dfrac{P(x)}{x-c}$

Possibilities: *Remainder Theorem: $P(c)=$ remainder of $x-c$

(a) 194

(b) $74x - 28$ plug "c" into polynomial $(x-(-3))$ "c" rewrite divisor in form $x-c$ in order to find c

(c) $x^2 - 1$

(d) -28

(e) -250

$P(-3)=(-3)^3-11(-3)^2+32(-3)-28$

$=-27-99-96-28$

$\boxed{=-250}$

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16. Suppose the graph of $y = f(x)$ is a parabola with vertex $(-1, 2)$ and goes through the points $(0, 6)$. Which of the following is an formula for $f(x)$?

h k

x y

Possibilities:

(a) $f(x) = 4(x + 2)^2 - 1$
(b) $f(x) = 4(x + 1)^2 + 2$
(c) $f(x) = (x - 1)^2 + 2$
(d) $f(x) = (x + 2)(x + 3)$
(e) $f(x) = (x + 1)(x + 6)$

*Use vertex form → $y = a(x-h)^2 + k$ & point to find formula

plug in values to solve for "a"

$6 = a(0+1)^2 + 2$

$4 = a$

use value for "a" and vertex point to complete formula

$\boxed{y = 4(x+1)^2 + 2}$

17. Solve for x.

$$6 \log_4 (x + 5) = 12$$

*rewrite in exponential form

$\log_a(y) = x \iff a^x = y$

divide away 6 first

Possibilities:

(a) $x = \sqrt[6]{12}$
(b) $x = \dfrac{12}{6\log(4)}$
(c) $x = 11$
(d) $x = -4.5$
(e) $x = 0$

$\log_4(x+5) = \frac{12}{6}$

$4^2 = x + 5$

rewrite in exponential form

and solve

$16 - 5 = x$

$\boxed{11 = x}$

18. Let $P(x) = 7x^{50} + 4x^{40} - 31x^{30} + 3x^{20} + 4$. List all possible rational zeros of $P(x)$ given by the Rational Zeros Theorem (but do not check to see which are actually zeros).

a_n a_0

*rational roots $\Rightarrow \dfrac{\text{factors of } a_0}{\text{factors of } a_n}$

Possibilities:

(a) $\pm 1, \pm 4, \pm 7/4$
(b) $\pm 1, \pm 1/2, \pm 1/4, \pm 7, \pm 7/2, \pm 7/4$
(c) $\pm 1, \pm 2, \pm 4, \pm 1/7, \pm 2/7, \pm 4/7$
(d) $\pm 1, \pm 2, \pm 4, \pm 7, \pm 7/2, \pm 7/4$
(e) $\pm 1, \pm 4, \pm 4/7$

$a_0 = 4 \Rightarrow$ factors: $\pm 1, \pm 2, \pm 4$

$a_n = 7 \Rightarrow$ factors: $\pm 1, \pm 7$

possible rational roots $\Rightarrow \pm\frac{1}{1}, \pm\frac{1}{7}, \pm\frac{2}{1}, \pm\frac{2}{7}, \pm\frac{4}{1}, \pm\frac{4}{7}$

$\boxed{\pm 1, \pm\frac{1}{7}, \pm 2, \pm\frac{2}{7}, \pm 4, \pm\frac{4}{7}}$

19. When a high school basketball team charges p dollars per ticket, the total revenue R from ticket sales is given by the formula

$$R(p) = 2160p - 120p^2.$$

What is the team's maximum revenue? $R(p) = -120p^2 + 2160p$

Possibilities:

*Maximums occur at vertex of quadratic

vertex $\Rightarrow (h, k) \Rightarrow \left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right)\right)$

Price for Max Revenue   MAX Revenue

(a) \$10360

(b) \$9

(c) \$8

(d) \$9720

(e) \$9980

$h \Rightarrow \frac{-2160}{2(-120)} = 9$

$k \Rightarrow R(9) = -120(9)^2 + 2160(9)$

$= -9720 + 19440$

$\boxed{= \$9720}$

20. Let $r(x) = \dfrac{x+4}{x+7}$. Find the asymptotes of r.

*VA occur at zeros in denominator & HA occur at ratio of leading terms

Possibilities:

(a) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = -4$.

(b) The vertical asymptote is $x = 1$ and the horizontal asymptote is $y = -7$.

(c) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = -7$.

(d) The vertical asymptote is $x = -4$ and the horizontal asymptote is $y = 1$.

(e) The vertical asymptote is $x = -7$ and the horizontal asymptote is $y = 1$.

VA: $x + 7 = 0$ $\boxed{x = -7}$

HA: $\frac{x}{x} = 1 \Rightarrow \boxed{y = 1}$ is H.A.

21. Explain how the graph of $g(x) = \underbrace{(x+5)}_{\text{affects input}}{}^2 \overbrace{- 8}^{\text{affects output}}$ is obtained from the graph of $f(x) = x^2$.

Input affects are counter-intuitive so, $\boxed{\text{left } 5}$

Possibilities:

(a) Shift the graph of f right 5 units and shift up 8 units to obtain the graph of g.

(b) Shift the graph of f left 5 units and shift down 8 units to obtain the graph of g.

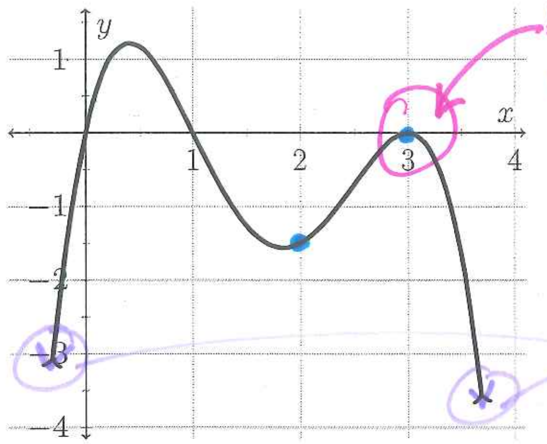
(c) Shift the graph of f right 5 units and shift down 8 units to obtain the graph of g.

(d) Shift the graph of f right 8 units and shift up 5 units to obtain the graph of g.

(e) Shift the graph of f left 8 units and shift down 5 units to obtain the graph of g.

output affects are intuitive → so, $\boxed{\text{down } 8}$

The next two problems refer to the graph shown. In the picture below, the graph of the polynomial function $P(x)$ is shown.



#23 graph touches at root, so even multiplicity.

both ends going down indicates negative leading coefficient.

both ends going in same direction indicates EVEN degree

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22. For the graph of the polynomial $P(x)$ drawn above, which of the following can you conclude about P?

*End behavior of graph determined by leading term

Possibilities:

(a) The degree of the polynomial is odd (even) and the leading coefficient is negative.

(b) The parity (even or odd) of the degree of the polynomial or the sign of the leading coefficient can not be determined by the graph. ← even & negative because both pointing down

(c) The degree of the polynomial is even and the leading coefficient is positive.

(d) The degree of the polynomial is odd and the leading coefficient is positive.

(e) The degree of the polynomial is even and the leading coefficient is negative.

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23. For the graph of the polynomial $P(x)$ drawn above, which of the following statements can be concluded?

*factors $(x-c) \iff P(c)=0 \iff x=c$ x-intercept

(I). $(x+1)$ is a factor of $P(x)$ No x-intercept @ $x=-1$

(II). When $P(x)$ is divided by $(x-2)$ the remainder is six. $P(2)\approx -1.5$ $P(2)\neq 6$

(III). $x=3$ is a root with even multiplicity.

*$P(c)$ = remainder

Possibilities:

(a) Only statements (I) and (II) are true.

(b) None of the statements are true.

(c) Only statement (III) is true.

(d) Only statement (II) is true.

(e) Statements (I), (II), and (III) are all true.

*roots with even multiplicity touch, while roots with odd multiplicity cross.

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$