MA109 — College Algebra Fall 2015
Exam 3 2015-11-18

Name: _ KEY _____ Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	11.	a	b	c	**d**	e
2.	a	b	c	**d**	e	12.	a	b	c	**d**	e
3.	a	b	**c**	d	e	13.	**a**	b	c	d	e
4.	a	**b**	c	d	e	14.	a	b	**c**	d	e
5.	a	**b**	c	d	e	15.	a	b	c	d	**e**
6.	a	**b**	c	d	e	16.	a	b	**c**	d	e
7.	a	b	c	d	**e**	17.	a	b	c	**d**	e
8.	a	b	c	d	**e**	18.	**a**	b	c	d	e
9.	a	**b**	c	d	e	19.	a	b	c	d	**e**
10.	a	b	**c**	d	e	20.	**a**	b	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the indicated value of the function when $x = \sqrt{6} + 2$.

$$f(x) = \sqrt{x+8} - x - 3$$

$f(\sqrt{6}+2) =$

$f(\sqrt{6}+2) = \sqrt{(\sqrt{6}+2)+8} - (\sqrt{6}+2) - 3$

$= \sqrt{\sqrt{6}+10} - \sqrt{6} - 2 - 3$

$= \boxed{\sqrt{\sqrt{6}+10} - \sqrt{6} - 5}$

Possibilities:

(a) $\sqrt{\sqrt{6}+10} - \sqrt{6} - 5$

(b) 5

(c) $\sqrt{10} - 5$

(d) $\sqrt{\sqrt{6}+10} - \sqrt{6} - 1$

(e) $\sqrt{16} - \sqrt{6} - 5$

2. Find $f(4)$ if $f(x) = \begin{cases} 8 & \text{if } x \leq 1 \\ 2x+6 & \text{if } 1 < x \leq 3 \\ 3x+3 & \text{if } 3 < x \leq 5 \\ 18 & \text{if } x > 5 \end{cases}$

$x = 4$

$\Leftarrow 3 < 4 \leq 5$

Possibilities:

$f(4) = 3(4) + 3$

$= 12 + 3$

$= \boxed{15}$

(a) 18

(b) 8

(c) 12

(d) 15

(e) 14

3. Find the domain of $\sqrt[3]{\frac{x-3}{7}}$

Possibilities:

(a) $(-\infty, 3) \cup (3, \infty)$

(b) $(3, \infty)$

(c) $(-\infty, \infty)$

(d) $[\frac{3}{7}, \infty)$

(e) $[3, \infty)$

odd roots have no domain restrictions

rational expressions have domain restrictions only for variable expressions in denominator

No domain restrictions due to odd root, nor due to rational expression, implies domain is all real numbers

$(-\infty, \infty)$

4. Find the domain of $\frac{3}{\sqrt{x-7}}$

Possibilities:

(a) $(-\infty, 7) \cup (7, \infty)$

(b) $(7, \infty)$

(c) $[\frac{3}{7}, \infty)$

(d) $[7, \infty)$

(e) $(-\infty, \infty)$

expressions under even must remain greater than or equal to zero

denominators of rational expressions must not equal zero

$x - 7 > 0$

$x > 7$

$(7, \infty)$

5. Which situation below is most reasonable depicted in this graph:

Possibilities:

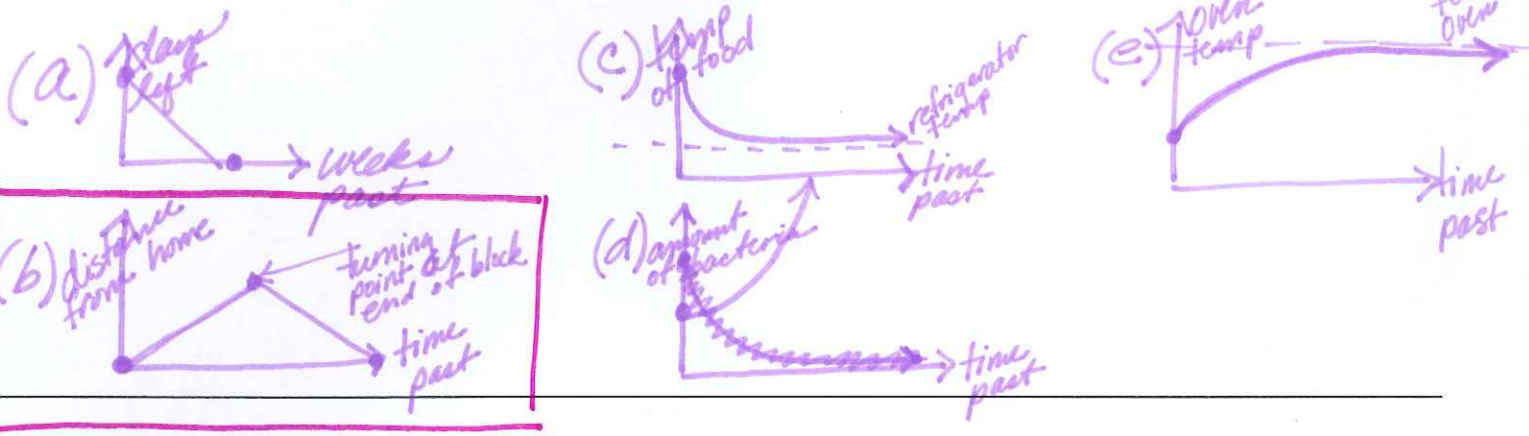
(a) y is the how many days are left in the semester after x weeks of school, if $x = 0$ is the first week of class.

(b) y is the distance from home at time x as you run to the end of the block and back at a steady pace.

(c) y is the temperature of left-over food at time x if the food is placed in the refrigerator at time $x = 0$.

(d) y is the number of bacteria at time x if the bacteria experience a steady rate of exponential growth.

(e) y is the temperature of an oven at time x if it switched on at time $x = 0$ and left on.



6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 10 to 15 seconds.

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	50	200	450	800	1250	1800

Possibilities:

(a) 20 feet per second

(b) 50 feet per second

(c) 80 feet per second

(d) 60 feet per second

(e) 30 feet per second

$$\text{Average speed} = \frac{\Delta \text{ distance} \leftarrow \text{feet}}{\Delta \text{ time} \leftarrow \text{seconds}}$$

$$= \frac{450 - 200}{15 - 10}$$

$$= \frac{250}{5}$$

$$= 50 \text{ feet per second}$$

7. Simplify the formula for the average rate of change of $f(x) = (x-3)^2 + 7$ from $x = 3$ to $x = 3+h$

Possibilities:

(a) $3 + 2h$

(b) $2h$

(c) $6 + h$

(d) 1

(e) h

$$A.R.O.C. = \frac{f(b) - f(a)}{b - a} \quad (a \to 3,\ b \to 3+h)$$

$$= \frac{f(3+h) - f(3)}{3+h-3}$$

$$= \frac{[(3+h)-3]^2 + 7 - [(3-3)^2 + 7]}{h}$$

$$= \frac{h^2 + 7 - 7}{h}$$

$$= \frac{h^2}{h}$$

$$= \boxed{h}$$

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8. Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 3x^2 + 7x + 8$ and $g(x) = 2x - 9$

Possibilities:

(a) $\left[\frac{9}{2}, \infty\right)$

(b) $\left[\frac{-7 \pm \sqrt{7^2 - 4(3)(8)}}{6}, \infty\right)$

(c) $(-\infty, \frac{2}{9})$

(d) $(-\infty, \infty)$

(e) $(-\infty, \frac{9}{2}) \cup (\frac{9}{2}, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2 + 7x + 8}{2x - 9}$$

Domain of f: $(-\infty, \infty)$

Domain of g: $(-\infty, \infty)$

Domain of $\frac{f}{g}$: $g(x) \neq 0$

$2x - 9 \neq 0$

$2x \neq 9$

$x \neq \frac{9}{2}$

$$\boxed{(-\infty, \tfrac{9}{2}) \cup (\tfrac{9}{2}, \infty)}$$

9. Find $(f-g)(6)$ where $f(x) = 4x^2 - 8x - 9$ and $g(x) = 3x - 2$

Possibilities:

(a) 103

(b) 71

(c) 259

(d) 67

(e) 887

$(f-g)(6) = f(6) - g(6)$

$= [4(6)^2 - 8(6) - 9] - [3(6) - 2]$

$= [4(36) - 48 - 9] - [18 - 2]$

$= 87 - 16$

$= \boxed{71}$

10. Simplify the formula for $(f \circ g)(x)$ if $f(x) = 1 - x$ and $g(x) = \frac{x-1}{x}$

Hint: try plugging in $x = 9$

Possibilities:

(a) $\frac{x}{x-1}$

(b) $9x$

(c) $\frac{1}{x}$

(d) x

(e) $\frac{9}{x}$

USING HINT:

$(f \circ g)(9) = f(g(9)) = f\left(\frac{9-1}{9}\right) = f\left(\frac{8}{9}\right)$

$= 1 - \frac{8}{9}$

$= \frac{1}{9}$

$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-1}{x}\right) = 1 - \frac{x-1}{x}$

$= \frac{x}{x} - \frac{x-1}{x}$

$= \frac{x - (x-1)}{x}$

$= \frac{x - x + 1}{x}$

$= \boxed{\frac{1}{x}}$

11. Suppose that the graph of $y = f(x)$ contains the point $(4, 8)$. Find a point that must be on the graph of $y = g(x)$ for $g(x) = 9 + f(3x + 2)$.

Possibilities:

(a) $\left(\frac{2}{3}, -1\right)$

(b) $(14, 17)$

(c) $\left(-\frac{2}{3}, -1\right)$

(d) $\left(\frac{2}{3}, 17\right)$

(e) $(14, -1)$

$(4, 8)$

$x_1 \longrightarrow 3x_2 + 2$

$4 \longrightarrow 3x_2 + 2$

$4 = 3x_2 + 2$

$2 = 3x_2$

$\frac{2}{3} = x_2$

$g(x_2)$

$9 + f(3x_2 + 2)$

$9 + f(3(\frac{2}{3}) + 2)$

$9 + f(4)$

$9 + 8$

17

12. Which sequence of transformations will transform the graph of the function f into the graph of the function g?

$f(x) = \sqrt{x} + 4 \qquad g(x) = \sqrt{x - 3} + 6$

shift up 2 units

shift right 3 units

Possibilities:

(a) shift right by 3 then shift down by 2

(b) shift left by 3 then shift up by 2

(c) shift left by 2 then shift down by 3

(d) shift right by 3 then shift up by 2

(e) shift left by 3 then shift down by 2

$f(x) = \sqrt{x} + 4$

shift right 3 $\Rightarrow f(x-3) = \sqrt{x-3} + 4$

shift up 2 $\Rightarrow f(x-3) + 2 = \sqrt{x-3} + 4 + 2$

$g(x) = \sqrt{x-3} + 6$

13. Use algebra to find the inverse of the given one-to-one function.

$$f(x) = \left(x^5 + 9\right)^4$$

Possibilities:

(a) $f^{-1}(x) = \sqrt[5]{\sqrt[4]{x} - 9}$

(b) $f^{-1}(x) = \sqrt[5]{\sqrt[9]{x} - 4}$

(c) $f^{-1}(x) = x^{20} + 9$

(d) $f^{-1}(x) = \sqrt[4]{\sqrt[5]{x} - 9}$

(e) $f^{-1}(x) = \left(x^4 + 9\right)^5$

$y = (x^5+9)^4$

$x = (y^5+9)^4$

$\sqrt[4]{x} = y^5 + 9$

$\sqrt[4]{x} - 9 = y^5$

$\sqrt[5]{\sqrt[4]{x} - 9} = y$

$\boxed{\sqrt[5]{\sqrt[4]{x} - 9} = f^{-1}(x)}$

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14. Use algebra to find the inverse of the given one-to-one function. $f(x) = \dfrac{4x}{8x+9}$

Possibilities:

(a) $f^{-1}(x) = \dfrac{1}{2}x + 9$

(b) $f^{-1}(x) = \dfrac{4x}{8x - 9}$

(c) $f^{-1}(x) = \dfrac{9x}{4 - 8x}$

(d) $f^{-1}(x) = \dfrac{8x + 9}{4x}$

(e) $f^{-1}(x) = \dfrac{9x}{4x + 8}$

$y = \dfrac{4x}{8x+9}$

$x = \dfrac{4y}{8y+9}$

$x(8y+9) = 4y$

$8xy + 9x = 4y$

$8xy - 4y = -9x$

$y(8x-4) = -9x$

$y = \dfrac{-9x}{8x-4}$

$y = \dfrac{-9x}{-(4-8x)}$

$y = \dfrac{9x}{4-8x}$

$f^{-1}(x) = \boxed{\dfrac{9x}{4-8x}}$

15. Write the given expression without using radicals.

$\sqrt[7]{x^5}$

$\sqrt[b]{x^a} = x^{a/b}$

$\sqrt[7]{x^5} = x^{5/7}$

Possibilities:

(a) $x^5 - x^7$

(b) x^2

(c) x^{-2}

(d) $x^{7/5}$

(e) $x^{5/7}$

16. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week:	1	2	3	4	5	6
Frogs:	45	135	405	1215	3645	10935

Which exponential growth model most closely matches the observations, if t is the week number?

Possibilities:

(a) $15\left(9^{(t/7)}\right)$

(b) $3\left(45^t\right)$

(c) $15\left(3^t\right)$

(d) $3\left(45^{(t/7)}\right)$

(e) $45\left(9^t\right)$

$f(t) = P_0 a^t$

$f(1) = P_0 a^1$

$45 = P_0 a$

$f(2) = P_0 a^2$

$\frac{135}{3} = \frac{P_0 a^2}{3}$

$45 = \frac{P_0 a^2}{3}$

$P_0 a = \frac{P_0 a^2}{3}$

$a = \frac{a^2}{3}$

$0 = \frac{a^2}{3} - a$

$0 = a^2 - 3a$

$0 = a(a-3)$ → $a = 0$ or $a - 3 = 0$, $a = 3$

$45 = P_0(3)$

$15 = P_0$

$f(t) = 15(3^t)$

17. Determine how much money (to the nearest cent) will be in a savings account if the initial deposit was \$2000 and the interest rate is 3.250% compounded continuously for 7 years.

Possibilities:

(a) \$2510.82

(b) \$2510.85

(c) \$2510.88

(d) \$2510.91

(e) \$2510.94

P_0 r t

$P(t) = P_0 e^{rt}$

$P(t) = 2000\left[e^{(.0325)(7)}\right]$

$P(t) = 2000\, e^{.2275}$

$P(t) = 2510.9148795\ldots$

$= \$2510.91$

18. Translate the given exponential statement into an equivalent logarithmic statement.

$$4^x = 8$$

Possibilities:

(a) $\log_4(8) = x$

(b) $\log_8(4) = x$

(c) $\log_8(x) = 4$

(d) $\log_x(4) = 8$

(e) $\log_4(x) = 8$

$a^r = s \iff \log_a(s) = r$

$4^x = 8$

$\log_4(8) = x$

19. Write the domain of the function $h(x) = \log(x-3)$ in interval notation.

Possibilities:

(a) $(-\infty, 3) \cup (3, \infty)$

(b) $(-\infty, -3)$

(c) $(-\infty, 3]$

(d) $(-\infty, \infty)$

(e) $(3, \infty)$

$x - 3 > 0$

$x > 3$

3

$(3, \infty)$

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20. Write the given expression as a single logarithm.

$$4\log(x) + \log(8y) - \log(9z)$$

$\log(x^4) + \log(8y) - \log(9z)$

$\log[(x^4)(8y)] - \log(9z)$

Possibilities:

(a) $\log\left(\frac{x^4(8y)}{9z}\right)$

(b) $\log\left(x^4y^8z^9\right)$

(c) $\log\left(\frac{x^4y^8}{z^9}\right)$

(d) $\log(4x + 8y - 9z)$

(e) $\log(4x(8+y) - 9 - z)$

$\log\left[\frac{(x^4)(8y)}{9z}\right]$

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$