detailed

MA109 — College Algebra Exam 2	Fall 2015 2015-10-21	Name: KEY	Sec.: ________

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	(a)	**(b)**	(c)	(d)	(e)	11.	(a)	**(b)**	(c)	(d)	(e)
2.	**(a)**	(b)	(c)	(d)	(e)	12.	(a)	(b)	**(c)**	(d)	(e)
3.	(a)	(b)	**(c)**	(d)	(e)	13.	(a)	(b)	(c)	(d)	**(e)**
4.	(a)	(b)	(c)	**(d)**	(e)	14.	(a)	(b)	(c)	**(d)**	(e)
5.	(a)	(b)	**(c)**	(d)	(e)	15.	**(a)**	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	**(e)**	16.	(a)	**(b)**	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	**(e)**	17.	(a)	(b)	**(c)**	(d)	(e)
8.	(a)	**(b)**	(c)	(d)	(e)	18.	(a)	(b)	(c)	**(d)**	(e)
9.	**(a)**	(b)	(c)	(d)	(e)	19.	**(a)**	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	**(e)**	20.	(a)	(b)	(c)	**(d)**	(e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: ______________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Use the substitution method to find all solutions of the system of equations.

① solve one equation for one variable
② substitute that expression for the same variable in other equation

$$x^2 + y = 15$$
$$x - y + 3 = 0$$

solve 1st equation for "y"
$y = \underbrace{15 - x^2}$

substitute into 2nd equation for "y"
$x - (15 - x^2) + 3 = 0$
$x - 15 + x^2 + 3 = 0$
$x^2 + x - 12 = 0$

Possibilities:

(a) $(x = 4, y = -1)$ and $(x = -4, y = 7)$
(b) $(x = -4, y = -1)$ and $(x = 3, y = 6)$
(c) $(x = 3, y = 6)$ only
(d) $(x = -4, y = -1)$ only
(e) No real solutions

③ solve new equation that now has only 1 variable

solve quadratic for "x"
$(x + 4)(x - 3) = 0$
$x + 4 = 0 \quad x - 3 = 0$

④ use solution for one variable to find second variable

$y = 15 - (-4)^2 = -1$
$y = 15 - (3)^2 = 6$

$\boxed{x = -4,\ y = -1}$ $\boxed{x = 3,\ y = 6}$

2. Suppose you are solving the system of equations below using the substitution method. You solve for y in the first equation and substitute it into the second equation. What equation must you solve then?

* solve 1st equation for "y"

$$4x^8 + y = 2$$
$$9x^3 + 5y = 6$$

$y = \underbrace{2 - 4x^8}$

$9x^3 + 5y = 6$

Possibilities:

(a) $9x^3 + 5\left(2 - 4x^8\right) = 6$

(b) $9\left(\sqrt[8]{2 - 4x^8}\right)^3 + 5y = 6$

(c) $9\left(2 - 4x^8\right)^3 + 5y = 6$

(d) $9x^3 + 5\left(\sqrt[8]{2 - y}\right) = 6$

(e) $9x^3 + 5\left(\sqrt[8]{2 - 4x^8}\right) = 6$

* substitute your solution for "y" into 2nd equation replacing "y" with its equivalent expression

$\boxed{9x^3 + 5(2 - 4x^8) = 6}$

3. Use the elimination method to solve the system. The multiple choice problem only asks you for y.

→ * "eliminate" the x-variable by adding an appropriate, equivalent multiple of second equation

$$\begin{aligned} 17x + 13y &= 103 \\ 17x + 12y &= 99 \end{aligned}$$

negate 2nd equation so coefficients of "x" are same but opposite signs

Possibilities:

(a) Every solution has $y = \frac{4}{17}$

(b) Every solution has $y = 3$

(c) Every solution has $y = 4$

(d) Every solution has $y = \frac{4}{13}$

(e) Every solution has $y = -13$

back to first equation

$$\begin{aligned} \cancel{17x} + 13y &= 103 \\ -\cancel{17x} - 12y &= -99 \\ \hline \boxed{y = 4} \end{aligned}$$

x variable is now "eliminated"

* this question does not require you to also solve for x!

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4. Use the elimination method to find all solutions of the system of equations.

* changing the sign of y^2 term will eliminate y variable, leaving x variable to solve for first.

$$\begin{cases} 9x^2 - 7y^2 = 81 \\ 8x^2 - 7y^2 = 65 \end{cases}$$

$$\begin{aligned} 9x^2 - \cancel{7y^2} &= 81 \\ -8x^2 + \cancel{7y^2} &= -65 \\ \hline x^2 &= 16 \\ \sqrt{x^2} &= \pm\sqrt{16} \\ x &= \pm 4 \end{aligned}$$

Possibilities:

(a) $(x = 4, y = 3)$ only

(b) $(x = -9, y = 7)$ and $(x = -8, y = 7)$

(c) $(x = 4, y = 3)$ and $(x = -4, y = -3)$

(d) $(x = 4, y = 3)$, $(x = -4, y = 3)$, $(x = 4, y = -3)$, and $(x = -4, y = -3)$

(e) $(x = 9, y = 7)$ and $(x = 8, y = 7)$

* use solutions for x to solve for y also

$$\boxed{x = 4}$$
$$\begin{aligned} 8(4)^2 - 7y^2 &= 65 \\ 8(16) - 7y^2 &= 65 \\ -7y^2 &= -63 \\ y^2 &= 9 \\ \boxed{y = \pm 3} \end{aligned}$$

$$\boxed{x = -4}$$
$$\begin{aligned} 9(-4)^2 - 7y^2 &= 81 \\ 9(16) - 7y^2 &= 81 \\ -7y^2 &= -63 \\ y^2 &= 9 \\ \boxed{y = \pm 3} \end{aligned}$$

5. Use the elimination method to find all solutions of the system of equations.

* easiest to eliminate y-variable 1st since one is a multiple of the other

$$\begin{cases} \dfrac{37}{x} + \dfrac{14}{y} = 113 \\ \dfrac{17}{x} + \dfrac{7}{y} = 54 \end{cases}$$

-2

multiplying by -2 will get same coefficient with opposite signs

$$\frac{37}{x} + \frac{14}{y} = 113$$
$$+\ \ -\frac{34}{x} - \frac{14}{y} = -108$$
$$x\left(\frac{3}{x} = 5\right)$$

*solve for x

clear denominator!

$$\frac{x}{1}\left(\frac{3}{x}\right) = 5x$$
$$3 = 5x$$
$$\boxed{3/5 = x}$$

Possibilities:

(a) $(x = 37, y = 14)$ and $(x = 17, y = 7)$

(b) $\left(x = \frac{3}{5}, y = \frac{3}{11}\right)$ and $\left(x = -\frac{3}{5}, y = -\frac{3}{11}\right)$

(c) $\left(x = \frac{3}{5}, y = \frac{3}{11}\right)$ only

(d) $(x = -37, y = 14)$ and $(x = -17, y = 7)$

(e) $(x = 37, y = 14)$, $(x = -37, y = 14)$, $(x = 17, y = 7)$, and $(x = -17, y = -7)$

* now substitute back x-solution into one of original equations to solve for y

$$\frac{17}{3/5} + \frac{7}{y} = 54$$
$$(17)(5/3) + \frac{7}{y} = 54$$
$$\frac{7}{y} = 54 - \frac{85}{3}$$
$$\frac{7}{y} = \frac{162-85}{3}$$
$$\left(\frac{7}{y} = \frac{77}{3}\right)3y$$
$$21 = 77y$$
$$\boxed{\frac{3}{11} \Leftarrow \frac{21}{77} = y}$$

6. The graph of two equations is shown below. Determine the number of solutions for the system of equations.

* solutions to systems are points of intersection of graphs!!

Possibilities:

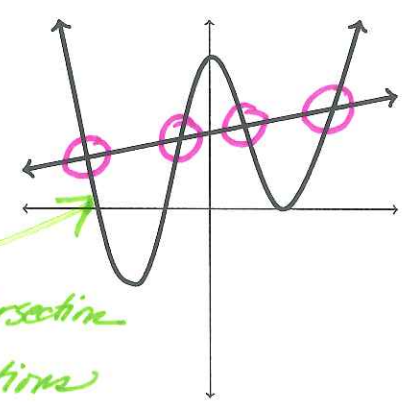
(a) 3

(b) 1

(c) 2

(d) 0

(e) 4



4 points of intersection means 4 solutions to the system

7. Use graphical approximation (a root finder or an intersection finder) to find a solution of the equation in the given interval. (Round your answer to four decimal places.)

$$x^5 + 4 = 8x^4; \quad (-\infty, 0]$$

* graph 2 equations

$y = x^5 + 4$

$y = 8x^4$

Possibilities:

(a) $x = -0.8250$

(b) $x = -0.8239$

(c) $x = -0.8228$

(d) $x = -0.8217$

(e) $x = -0.8206$

* look for intersection point to the left of y-axis ... this is the interval $(-\infty, 0]$

* then record x-value

OR * graph 1 equation

$y = x^5 - 8x^4 + 4$

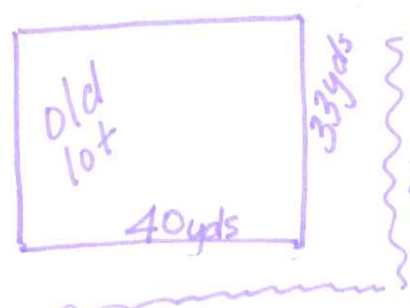
* look for x-intercept (when $y=0$) that is to the left of y-axis

* record x-value

8. A corner lot has dimensions 40 by 33 yards. The city plans to take a strip of uniform width along the two sides bordering the streets to widen these roads. How wide should the strip be if the remainder of the lot is to have an area of 980 square yards?

Which equation should you solve in order to find the answer? The variable x represents the width of the strip in yards.

* find expressions for length & width of new corner lot after strip has been removed



Possibilities:

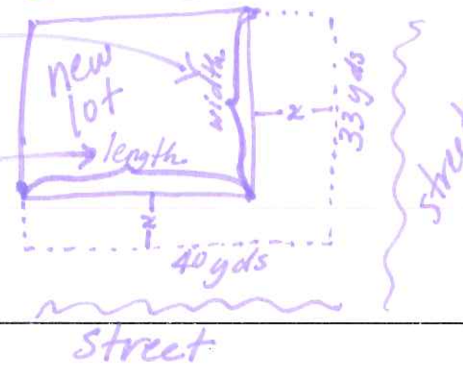
(a) $(40)(33) - x^2 = 980$

(b) $(40 - x)(33 - x) = 980$

(c) $x^2 = 980$

(d) $x = 1320 - 980$

(e) $(40)(33) = x$



new area $= 980$

$l \cdot w = 980$

$$\boxed{(40-x)(33-x) = 980}$$

length: $40-x$

width: $33-x$

9. You have already invested \$400 in a stock with an annual return of 10%. How much of an additional \$1,400 should be invested at 20% and how much at 5% so that the total return on the entire \$1,800 is 15%?

What equations should be solved if x is the amount of money invested at 20% and y is the amount of money invested at 5%?

* find system of equations where one equation represents total money invested in 3 accounts

Possibilities:

(a) $\left\{\begin{array}{rcrcrcr} 400 & + & x & + & y & = & 1800 \\ .10(400) & + & .20x & + & .05y & = & .15(1800) \end{array}\right\}$

(b) $\left\{\begin{array}{rcrcrcr} .15 & + & x & + & y & = & 1800 \\ 1400 & + & .20x & + & .05y & = & .10(400) \end{array}\right\}$

(c) $\left\{\begin{array}{rcr} x & = & .20(1400) \\ y & = & .05(400) \end{array}\right\}$

(d) $\left\{\begin{array}{rcrcr} .05x & + & .20y & = & .10(1400) \\ .20x & + & .05y & = & .15(1800) \end{array}\right\}$

(e) $\left\{\begin{array}{rcrcr} x & + & y & = & 400 \\ .20x & + & .05y & = & .15(1800) \end{array}\right\}$

amount in 10% account | amt in 20% acct | amt in 5% acct. | TOTAL investment

$\boxed{400 + x + y = 1800}$

* and other equation represents the total amount of interest earned ... (Interest = (rate)(principal)) expected to know formula!

interest from 10% account | interest from 20% account | interest from 5% account | TOTAL interest

$\boxed{.10(400) + .20(x) + .05(y) = .15(1800)}$

10. A concrete walk of uniform width is to be built around a giant circular pool. The radius of the pool is 14 meters, and enough concrete is available to cover 53.64π square meters (approximately). If all the concrete is to be used, how wide should the walk be (approximately)? Choose the closest answer.

* find equation expressing relationship between area of pool & area of walkway

expected to know formula!

Area of pool & walkway − Area pool = Area walkway

Possibilities:

(a) 6.68 meters wide

(b) 3.83 meters wide

(c) 39.6 meters wide

(d) 14 meters wide

(e) 1.8 meters wide

$r_{pool} = 14$

$r_{walkway} = x$

$r_{pool \,\&\, walkway} = 14 + x$

Area of Circle πr^2

$\pi(14+x)^2 - \pi(14)^2 = 53.64\pi$

divide by π & expand expressions

$14^2 + 28x + x^2 - 14^2 = 53.64$

14^2 subtracts away

$x^2 + 28x - 53.64 = 0$

use quadratic formula $a=1$ $b=28$ $c=-53.64$

$x = \dfrac{-28 \pm \sqrt{28^2 - 4(1)(53.64)}}{2(1)}$

$x = \dfrac{-28 + \sqrt{998.56}}{2}$ $\quad x = \dfrac{-28 - \sqrt{998.56}}{2}$

$x \approx 1.8$ meters $\quad$ ~~$x \approx -29.8$ meters~~ ← negative answer is non-sensical regarding distance

11. Find the equilibrium price. In the supply and demand equations, p is price (in dollars) and x is quantity (in thousands). Please round your answer to the nearest hundredth (the nearest cent).

*equilibrium price occurs when prices are equal

$$\begin{array}{lcrcr} \text{Supply: } p & = & 6x & - & 3 \\ \text{Demand: } p & = & -9x & + & 5 \end{array}$$

Possibilities:

(a) $p = \$0.53$

(b) $p = \$0.20$

(c) $p = \$2$

(d) $p = \$7.50$

(e) $p = \$3$

Supply p = Demand p

$6x - 3 = -9x + 5$

$15x = 8$

$x = 8/15$

*equilibrium price occurs when quantity, $x = 8/15$ (in thousands).

now find p, when $x = 8/15$

$p = -9(8/15) + 5$

$= \frac{-72}{15} + \frac{75}{15}$

$= \frac{3}{15} \Rightarrow .2 \Rightarrow \boxed{\$.20}$

12. A radiator contains 6 quarts of fluid, 25% of which is antifreeze. How much fluid should be drained and replaced with pure (100%) antifreeze so that the new mixture is 55% antifreeze?

Possibilities: *define variable → x = amount to be drained, y = amount to be kept

(a) 7.2 quarts drained and replaced

(b) 3.3 quarts drained and replaced

(c) 2.4 quarts drained and replaced

(d) 1.5 quarts drained and replaced

(e) 6 quarts drained and replaced

*establish system to solve

amount of fluid → $x + y = 6$

amount of antifreeze → $1.00(x) + .25(y) = .55(6)$

* solve the system (preferably for variable representing amount to be drained to avoid having to go back & substitute)

$(x + y = 6)(-.25) \Rightarrow -.25x - .25y = -.25(6)$

$+ \quad 1.00x + .25y = .55(6)$

$\frac{.75x}{.75} = \frac{1.8}{.75}$

$\boxed{x = 2.4 \text{ qts.}}$

13. Solve the inequality and express your answer as simplified inequalities.

$$4x + 8 \leq 9x + 3$$

*LINEAR inequalities can be solved much like LINEAR equations with the exception that multiplying or dividing by negative number reverses the inequality symbol

Possibilities:

(a) $x \geq -1$

(b) $x \leq -2$

(c) $x \leq -1$

(d) $x \leq 1$

(e) $x \geq 1$

$4x + 8 \leq 9x + 3$

$-9x \qquad -9x$

$-5x + 8 \leq 3$

$-8 \qquad -8$

$\frac{-5x}{-5} \leq \frac{-5}{-5}$

$\boxed{x \geq 1}$

---

14. Solve the inequality. Express your answer in interval notation.

$$\left|3 + \frac{1}{3}x\right| \leq \frac{4}{3}$$

* absolute value inequalities must account for both positive and negative values within absolute value

can be both + or −

Possibilities:

(a) $[0, 13]$

(b) $[5, 13]$

(c) $[0, \frac{4}{3}]$

(d) $[-13, -5]$

(e) $[\frac{4}{3}, 3]$

*once each possibility is solved separately what values satisfy BOTH solutions

find overlap (in this case)

$+(3 + \frac{1}{3}x) \leq \frac{4}{3}$

$3 + \frac{1}{3}x \leq \frac{4}{3}$

$\frac{1}{3}x \leq \frac{4}{3} - 3$

$3\left(\frac{1}{3}x\right) \leq \left(-\frac{5}{3}\right)3$

$x \leq -5$

AND

$-(3 + \frac{1}{3}x) \leq \frac{4}{3}$

$3 + \frac{1}{3}x \geq -\frac{4}{3}$

$\frac{1}{3}x \geq -\frac{4}{3} - 3$

$3\left(\frac{1}{3}x\right) \geq \left(-\frac{13}{3}\right)3$

$x \geq -13$

$x \geq -13$

$x \leq -5$

$-13 \qquad -5$

$\boxed{[-13, -5]}$

15. Solve the inequality. Answer in interval notation.

$$\frac{x-4}{x-8} \le 0$$

*Rational inequalities require examination of sign changes for both numerator and denominator

sign changes occur on either side of 0

$x-4=0$
$x=4$

$x-8=0$
$x=8$

Possibilities:

(a) $[4,8)$

(b) $(-\infty,4]$

(c) $(-\infty,8]$

(d) $(-\infty,4]\cup[8,\infty)$

(e) $[4,8]$

cannot include 8 here b/c $\frac{4}{0}$ is undefined

can include 4 b/c $\frac{0}{4}=0$

* separate the number line into regions where signs have potential to change

test values in each region

$x=1$ $\frac{x-4}{x-8} \Rightarrow \frac{-}{-}$ $\oplus$

$x=6$ $\frac{x-4}{x-8} \Rightarrow \frac{+}{-}$ $\ominus$

$x=10$ $\frac{x-4}{x-8} \Rightarrow \frac{+}{+}$ $\oplus$

*choose regions satisfying inequality

in this case negative or $=0$

16. Solve the inequality. Answer by choosing the correct number line.

$0 \le (x-4)(x-3)^2$

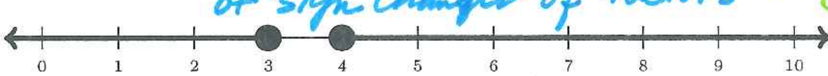
*NON-LINEAR inequalities require same examination of sign changes of factors

$x-4=0$
$x=4$

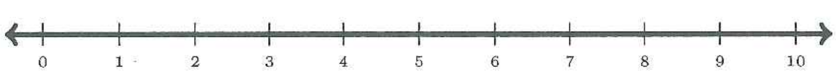
$x-3=0$
$x=3$

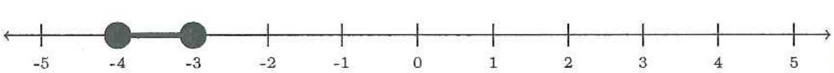
zeros both included b/c can be equal to 0

Possibilities:

(a)


(b)


(c)


(d)


(e)


$x=0$ $x-4 \Rightarrow -$ $(x-3)^2 \Rightarrow +$ $\ominus$

$x=3.5$ $x-4 \Rightarrow -$ $(x-3)^2 \Rightarrow +$ $\ominus$

$x=7$ $x-4 \Rightarrow +$ $(x-3)^2 \Rightarrow +$ $\oplus$

* choose & test values in each region to determine whether expression will be + or − in each region

*choose regions satisfying inequality

in this case positive or $=0$

17. Solve the inequality. Answer by choosing the correct number line.

$$0 < \frac{(x-4)(x-8)}{(x-9)(x-3)}$$

positive

* rational inequalities require examination of sign changes in numerator & denominator

sign changes occur at any potential 0

$x-4=0$
$x=4$

$x-8=0$
$x=8$

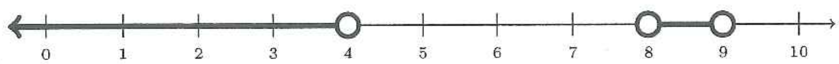
$x-9=0$
$x=9$

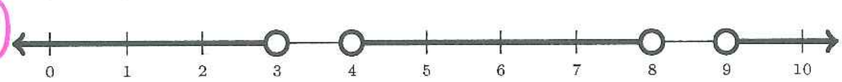
$x-3=0$
$x=3$

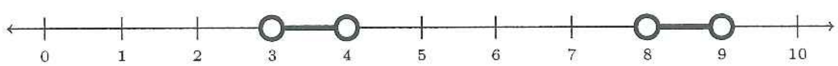
none of these zeros are included b/c expression $\neq 0$

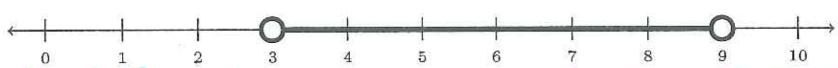
Possibilities:

(a) 

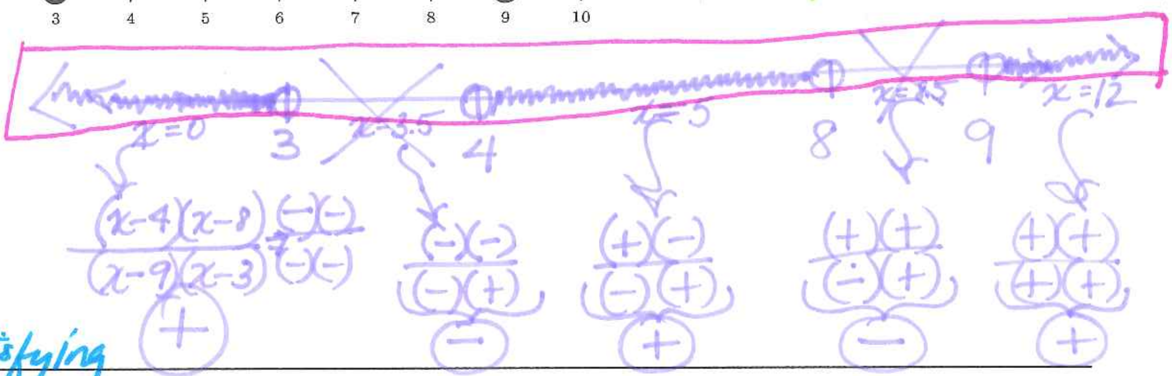
(b) 

(c) 

(d) 

(e) 

* Separate into regions
* determine sign of each region
* choose regions satisfying inequality



18. Find $f(3)$ from the graph of $y = f(x)$.

* $x=3$

y-value

* $f(3)$ asks what is function when $x=3$

Possibilities:

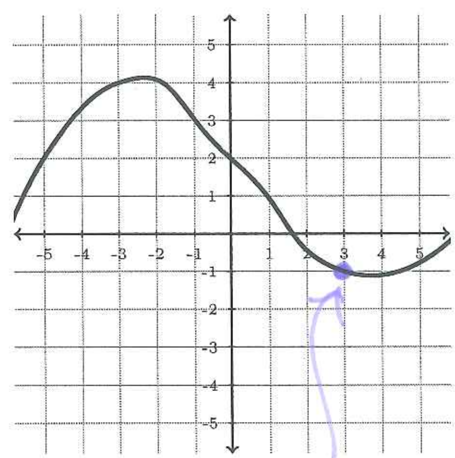
(a) $f(3) = 7$

(b) $f(3) = 3$

(c) $f(3) = 2$

(d) $f(3) = -1$

(e) $f(3) = 0$



$x=3$, $y=f(x)=-1 \Longrightarrow (3,-1)$

$\boxed{f(3)=-1}$

So, function is -1 when $x=3$

19. Find the indicated value of the function when $x = \sqrt{6} + 2$.

* value for x

sub in for x

$$f(x) = \sqrt{x+8} - x - 3$$

$f(\sqrt{6}+2) =$

$$f(\sqrt{6}+2) = \sqrt{(\sqrt{6}+2)+8} - (\sqrt{6}+2) - 3$$

Possibilities:

(a) $\sqrt{\sqrt{6}+10} - \sqrt{6} - 5$

(b) 5

(c) $\sqrt{10} - 5$

(d) $\sqrt{\sqrt{6}+10} - \sqrt{6} - 1$

(e) $\sqrt{16} - \sqrt{6} - 5$

* simplify

distribute negative

$$= \sqrt{\sqrt{6}+2+8} - \sqrt{6} - 2 - 3$$

combine like terms

$$= \boxed{\sqrt{\sqrt{6}+10} - \sqrt{6} - 5}$$

20. Let $f(x) = 4x^2 + 8$. Find $\dfrac{f(x+h) - f(x)}{h}$ if $h \neq 0$. Simplify your answer.

$$\frac{f(x+h) - f(x)}{h}$$

Possibilities:

(a) 16

(b) $\dfrac{h+16}{h}$

(c) $\dfrac{4h^2+8}{h}$

(d) $8x + 4h$

(e) $4x + 8h$

* Difference quotient requires finding $f(x+h)$ by plugging $(x+h)$ in for x in function

$$\frac{[4(x+h)^2+8] - [4x^2+8]}{h}$$

expand binomial

$$\frac{4(x^2+2xh+h^2)+8-4x^2-8}{h}$$

* Then simplify

distribute & combine like terms

$$\frac{\cancel{4x^2} + 8xh + 4h^2 \cancel{+8} \cancel{-4x^2} \cancel{-8}}{h}$$

divide h factor out away

$$\frac{\cancel{h}(8x+4h)}{\cancel{h}} \Longrightarrow \boxed{8x+4h}$$