

- This is a two hour exam. Clearly print your name on the first page and the top of the third page (second piece of paper). No books, notes, internet connection, or cell phone can be used during this exam. Any scratch paper must be provided to you by the proctor and turned in with the exam. A calculator maybe used; however, the calculator cannot have a Computer Algebra System (CAS) or a QWERTY keyboard. When you have completed the exam:

- 1) Turn in the entire exam (including cover page, and any scratch papers) to the proctor
- 2) Show your ID to the proctor
- 3) Sign the "Sign Out Sheet"

- All answers must be fully filled in on the front page, like so:



- The exam is out of 100 total points and includes 25 questions. Each question is 5 points and there are 5 extra credit questions. Only this front page will be graded and no partial credit will be awarded. Consequently, please double check to make sure that you have marked the answer you desire. A list of useful formulae and scrap paper is provided on the last page of the exam. Good Luck!

#1 (A) (B) (C) (D) (E)

#2 (A) (B) (C) (D) (E)

#3 (A) (B) (C) (D) (E)

#4 (A) (B) (C) (D) (E)

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#23 (A) (B) (C) (D) (E)

#24 (A) (B) (C) (D) (E)

#25 (A) (B) (C) (D) (E)

Name (Print): _____

Section Number: _____

Section	Instructor	Class Start Time	Exam Location
001	Drew Butcher	MWF 8:00 AM	MEH
002	Konstantina Christodouloupoulou	MWF 9:00 AM	CB 106
003	Konstantina Christodouloupoulou	MWF 2:00 PM	CB 118
004	Drew Butcher	MWF 11:00 AM	MEH
005	Drew Butcher	MWF 3:00 PM	MEH (A-K) & MDS 220 (L-Z)
006	Jonathan Constable	TR 8:00 AM	MDS 223
007	Stephen Deterding	TR 8:00 AM	CB 110
008	Jonathan Constable	TR 9:30 AM	MDS 223
009	Stephen Deterding	TR 9:30 AM	CB 110
010	Michael Gustin	TR 11:00 AM	CB 114
011	Robert Davis	TR 11:00 AM	Worsham
012	Michael Gustin	TR 12:30 PM	CB 114
013	Robert Davis	TR 12:30 PM	Worsham
014	Ray Kremer	TR 2:00 PM	Worsham
015	Clinton Hines	TR 2:00 PM	Worsham
016	Ray Kremer	TR 3:30 PM	Worsham
017	Clinton Hines	TR 3:30 PM	Worsham

UK: "Go CATS"

Name: _____

Section: _____

1. (5 points) Determine the vertex of the parabola $y = -2x^2 - 20x - 57$ and state whether it opens upward or downward.

- A. vertex (5, -5); opens downward.
- B. vertex (-5, -7); opens downward.
- C. vertex (-5, -7); opens upward.
- D. vertex (5, -207); opens upward.
- E. vertex (5, -207); opens downward.

$a = -2, b = -20, c = -57$
 Vertex: $x = \frac{-b}{2a} = \frac{-(-20)}{2(-2)} = \frac{20}{-4} = -5$
 Not A, D, E.
 a is negative so it opens downwards
 like $y = -x^2$ B

2. (5 points) Find the rule of the quadratic function $f(x)$ whose graph has its vertex at (-2, 3) and passes through (0, 15).

- A. $f(x) = 3(x - 2)^2 + 3$
- B. $f(x) = 3(x + 2)^2 + 3$
- C. $f(x) = 3(x + 2)^2 + 5$
- D. $f(x) = (x + 2)^2 + 3$
- E. None of the above.

$y = a(x - h)^2 + k$ has vertex (h, k)
 so B, D, or E
 Check which passes through (0, 15)
 $y = a(x - h)^2 + k$
 $15 = a(0 + 2)^2 + 3$
 $15 = 4a + 3$
 $4a = 15 - 3 = 12$
 $a = 3$
 $y = 3(x + 2)^2 + 3$ B

3. (5 points) A salesperson finds that her sales average 55 cases per store when she visits 15 stores a week. Each time she visits an additional store per week, the average sales per store decreases by 1 case. How many stores should she visit each week if she wants to maximize her sales?

- A. 16 stores.
- B. 20 stores.
- C. 24 stores.
- D. 30 stores.
- E. 35 stores.

Cases per store	Stores per week	Cases per week
55	15	$55(15) = 825$
$55 - 1 = 54$	$15 + 1 = 16$	$54(16) = 864$
$55 - x$	$15 + x$	$(55 - x)(15 + x)$

Adding is good but how much?
 Where is the max of $y = (55 - x)(15 + x) = 825 + 40x - x^2$, Vertex $x = \frac{-40}{2(-1)} = 20$, $15 + 20 = 35$ stores

4. (5 points) Use algebraic, graphical, or numerical methods to find the real solution of the equation $4x^3 - 5x^2 + 3x - 5 = 0$. Approximate your answer to four decimal places.

- A. $x \approx 1.3641$
- B. $x \approx 1.3611$
- C. $x \approx 1.3621$
- D. $x \approx 1.3691$
- E. $x \approx 1.3631$

Use "Zero" or "root" function on calculator.
 Or plug each x into $4x^3 - 5x^2 + 3x - 5$
 and choose the one that gives an answer closest to 0.

5. (5 points) Describe the end behavior of the graph $y = -6x^{15} + 3x^8 + 2x - 14$.

- A. $y \rightarrow \infty$ as $x \rightarrow \infty$; $y \rightarrow \infty$ as $x \rightarrow -\infty$. \cup
- B. $y \rightarrow \infty$ as $x \rightarrow \infty$; $y \rightarrow -\infty$ as $x \rightarrow -\infty$. \swarrow
- C. $y \rightarrow -\infty$ as $x \rightarrow \infty$; $y \rightarrow \infty$ as $x \rightarrow -\infty$. \swarrow
- D. $y \rightarrow -\infty$ as $x \rightarrow \infty$; $y \rightarrow -\infty$ as $x \rightarrow -\infty$. \cup
- E. None of the above.

Same end behavior as $y = -x^{15}$
 odd, so / or
 neg so

6. (5 points) Find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x) = x^5 - x^3 + x - 8$ is divided by $x - 3$.

- A. $Q(x) = x^4 + 2x^3 + 3x^2 + 6x + 13$, $R(x) = 25$
- B. $Q(x) = x^4 - x^3 - x^2 + x + 3$, $R(x) = 0$
- C. $Q(x) = x^4 + 3x^3 + 8x^2 + 24x + 73$, $R(x) = 211$
- D. $Q(x) = x^4 + x^3 - 3x^2 - 6x + 15$, $R(x) = 211$
- E. None of the above.

$x-3 \overline{) x^5 - x^3 + x - 8}$ rem 211
 $x^4 - 3x^4$
 $3x^4 - x^3$
 $3x^4 - 9x^3$
 $8x^3$
 $8x^3 - 24x^2$
 $24x^2 + x$
 $24x^2 - 72x$
 $73x - 8$
 $73x - 219$
 211

7. (5 points) Let $f(x) = \frac{2x + 3}{x^2 + 5x + 6}$. Find the asymptotes of $f(x)$.

- A. The horizontal asymptote is $y = 0$ and the vertical asymptotes are $x = -2$ and $x = -3$.
- B. The horizontal asymptote is $y = 2$ and the vertical asymptotes are $x = -2$ and $x = -3$.
- C. The horizontal asymptote is $y = -\frac{3}{2}$ and the vertical asymptote is $x = -2$.
- D. The horizontal asymptote is $y = -\frac{3}{2}$ and the vertical asymptotes are $x = 0$ and $x = 2$.
- E. The horizontal asymptote is $y = 0$ and the vertical asymptotes are $x = 2$ and $x = 3$.

$f(x) = \frac{2x+3}{(x+2)(x+3)}$

vertical asymptotes at $x = -2$ and $x = -3$
 horizontal asymptote $y = \frac{0}{1} = 0$

8. (5 points) Solve the equation

- A. $\frac{9 \pm \sqrt{57}}{6}$
- B. $\frac{9 \pm \sqrt{105}}{6}$
- C. $\frac{-9 \pm \sqrt{57}}{6}$
- D. $-9 \pm \sqrt{57}$
- E. None of the above.

$$3x^2 = 9x - 2$$

$$3x^2 - 9x + 2 = 0$$

$$a=3, b=-9, c=2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{9 \pm \sqrt{81 - 24}}{6} \quad A$$

9. (5 points) Solve the equation $\sqrt{-x-7} = x+7$ for x .

- A. $x = -8$.
- B. $x = -7$.
- C. $x = -8$ and $x = -7$.
- D. All real numbers.
- E. No solutions.

$$-x-7 = (x+7)^2$$

$$-x-7 = x^2 + 14x + 49$$

$$0 = x^2 + 15x + 56$$

$$0 = (x+7)(x+8)$$

$$x = -7 \text{ and } x = -8$$

But!

Check: $x = -7, \sqrt{-(-7)-7} = \sqrt{0} = 0$
 $x+7 = -7+7 = 0 \checkmark$

$x = -8, \sqrt{-(-8)-7} = \sqrt{1} = 1$
 $-8+7 = -1 \neq 1$ ❌

B only

√ are not negative, $x+7$ is not negative.
 $x = -8$ is bad

10. (5 points) Find the equation of the circle with center $(-1, 2)$ and radius 7.

- A. $(x+1)^2 + (y-2)^2 = 49$.
- B. $(x-1)^2 + (y-2)^2 = 49$.
- C. $(x-1)^2 + (y+2)^2 = 7$.
- D. $(x-1)^2 + (y-2)^2 = 7$.
- E. $(x-1)^2 + (y+2)^2 = 25$.

$$(x-h)^2 + (y-k)^2 = R^2$$

$$(x+1)^2 + (y-2)^2 = 7^2$$

A

11. (5 points) Find the equation of the line through the points $(9, 6)$ and $(-1, 1)$

- A. $y = 2x + 3$.
- B. $y = 2x - 6$.
- C. $y = 2x - 3$.
- D. $y = \frac{1}{2}x - 6$.
- E. $y = \frac{1}{2}x + \frac{3}{2}$.

$$m = \frac{6-1}{9-(-1)} = \frac{5}{10} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(-1) + b \quad (\text{Plug in } (-1, 1))$$

$$b = 1.5$$

$$y = \frac{1}{2}x + 1.5 \quad E$$

12. (5 points) Find all solutions to the following system of equations.

$$\begin{aligned} x + 3y &= 10 \\ xy &= 7 \end{aligned}$$

$$x = 10 - 3y$$

$$(10 - 3y)y = 7$$

$$10y - 3y^2 = 7$$

$$0 = 3y^2 - 10y + 7$$

$$0 = (3y - 7)(y - 1)$$

$$y = \frac{7}{3} \quad y = 1$$

$$x = 10 - 3\left(\frac{7}{3}\right)$$

$$x = 10 - 3(1)$$

$$x = 3$$

$$x = 7$$

$$\left(3, \frac{7}{3}\right)$$

$$(7, 1)$$

D

A. (7, 1)

B. $\left(\frac{2}{3}, 3\right)$.

C. (1, 7) and $\left(3, \frac{7}{3}\right)$.

D. (7, 1) and $\left(3, \frac{7}{3}\right)$.

E. (1, 7) and $\left(\frac{7}{3}, 3\right)$.

13. (5 points) You have already invested \$400 in a stock with an annual return of 11%. How much of an additional \$1,500 should be invested at 12% and how much at 6% so that the total return on the entire \$1,900 is 9%? Round your answer to the nearest cent.

A. \$630 at 12% and \$870 at 6%

B. \$640.33 at 12% and \$859.67 at 6%

C. \$605.75 at 12% and \$894.25 at 6%

D. \$616.67 at 12% and \$883.33 at 6%

E. \$650 at 12% and \$850 at 6%

Already	Additional	Total	
400	+ x + y	= 1900	Invested
(0.11)400	+ 0.12x + 0.06y	= (0.09)1900	Return

$$y = 1900 - 400 - x = 1500 - x$$

$$(0.11)400 + 0.12x + 0.06(1500 - x) = (0.09)1900$$

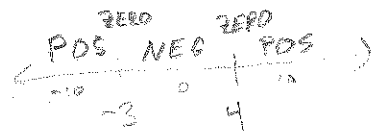
$$(0.12 - 0.06)x = \frac{(0.09)1900 - (0.11)400 - (0.06)1500}{0.06}$$

so $x = \frac{\dots}{(0.12 - 0.06)}$
 $x = \$616.67$
 $y = \$883.33$

14. (5 points) Solve the inequality and express your answer in interval notation.

$$(x + 3)(x - 4) \geq 0.$$

- A. $(-\infty, -3] \cup [4, \infty)$
- B. $(-\infty, -3) \cup (4, \infty)$
- C. $(-3, 4)$
- D. $[-3, 4]$
- E. $(-\infty, \infty)$



15. (5 points) Find the inverse of the one-to-one function

$$f(x) = \frac{x-5}{x}$$

- A. $f^{-1}(x) = \frac{x}{5-x}$
- B. $f^{-1}(x) = \frac{5}{x+1}$
- C. $f^{-1}(x) = 5(x-1)$
- D. $f^{-1}(x) = \frac{5}{1-x}$
- E. None of the above

$$y = \frac{x-5}{x}$$

$$x = \frac{y-5}{y} \quad (\text{switch } x \text{ and } y, \text{ then solve for } y)$$

$$xy = y-5$$

$$5 = y - xy$$

$$5 = y(1-x)$$

$$y = \frac{5}{1-x} \quad \text{D}$$

Check it: $f(10) = \frac{10-5}{10} = \frac{5}{10} = \frac{1}{2}$
 $f^{-1}(\frac{1}{2}) = \frac{5}{1-\frac{1}{2}} = 2(5) = 10$

16. (5 points) Let $f(x) = 7x^2 + 3x + 1$. Find $\frac{f(x+h) - f(x)}{h}$.

- A. 7
- B. 1
- C. $14x + 7h + 3$
- D. $14x + 3$
- E. None of the above.

$$\frac{[7(x+h)^2 + 3(x+h) + 1] - [7x^2 + 3x + 1]}{h} = \frac{7x^2 + 14xh + 7h^2 + 3x + 3h + 1 - 7x^2 - 3x - 1}{h}$$

$$= \frac{14xh + 7h^2 + 3h}{h} = \frac{h(14x + 7h + 3)}{h} = 14x + 7h + 3 \quad \text{C}$$

17. (5 points) Let $f(x) = \frac{x-5}{x^2+6x+8}$. Find the domain of $f(x)$.

- A. $(-4, -2)$
- B. $(-\infty, -4) \cup (-2, \infty)$
- C. $(-\infty, -4) \cup (-4, -2) \cup (-2, \infty)$
- D. $(-\infty, -4] \cup [-2, \infty)$
- E. $(-\infty, \infty)$.

$$y = \frac{x-5}{(x+2)(x+4)}$$

Don't divide by zero. $x = -2, x = -4$

C

18. (5 points) Explain how the graph of $g(x) = (x+3)^2 + 7$ is obtained from the graph of $f(x) = x^2$.

- A. Shift left 3 units and shift up 7 units.
- B. Shift right 3 units and shift up 7 units.
- C. Shift left 7 units and shift down 3 units.
- D. Shift left 3 units and shift down 7 units.
- E. Shift right 3 units and shift down 7 units.

Vertex moves from $(0,0)$ to $(-3,7)$

19. (5 points) Let $g(x) = \sqrt{3x+5}$ and $f(x) = x^2 - 1$. Find $g(f(x))$.

- A. $3\sqrt{x^2-1} + 5$
- B. $3\sqrt{x^2+2}$
- C. $3x^2 + 2$
- D. $\sqrt{3x^2+2}$
- E. $3x - 2$

X \xrightarrow{f} $x^2 - 1$

X \xrightarrow{g} $\sqrt{3x^2+5}$

X \xrightarrow{f} $x^2 - 1 \xrightarrow{g}$ $\sqrt{3(x^2-1)+5}$

$$g(f(x)) = g(x^2-1) = \sqrt{3(x^2-1)+5}$$

$$= \sqrt{3(x^2-1)+5} = \sqrt{3x^2-3+5} = \sqrt{3x^2+2}$$

D

20. (5 points) Determine how much money will be in a savings account if the initial deposit was \$2,000 and the interest rate is 7% compounded annually for 5 years. Round your answer to the nearest cent.

- A. 2829.56
- B. 2835.25
- C. 2837.47
- D. 2805.10
- E. 2838.04

$$2000 \left(1 + \frac{0.07}{1}\right)^{5(1)} = 2000(1.07^5)$$

= D

21. (5 points) Find the remainder of the division problem $\frac{x^5 - 3}{x - 2}$

- A. 240.
- B. 2.
- C. 29.
- D. 0.
- E. 3.

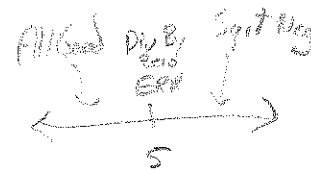
long division: $x-2 \overline{) x^5 - 3}$
 $x^5 - 2x^4$
 $2x^4$
 ...

easier: rem is $2^5 - 3 = 32 - 3 = 29$

22. (5 points) Find the domain of $f(x) = \frac{1}{\sqrt{5-x}}$.

- A. $(-\infty, 5)$
- B. $(-\infty, 5]$
- C. $(5, \infty)$
- D. $[5, \infty)$
- E. $(-\infty, 5) \cup (5, \infty)$

Don't divide by 0: $\sqrt{5-x} \neq 0$
 Don't square root neg: $5-x > 0$
 so $x < 5$
 A



23. (5 points) Suppose that you invest \$1000 at an interest rate of 5% compounded continuously. How long will it take for your investment to triple? Round your answer to the nearest year.

- A. About 18 years.
- B. About 19 years.
- C. About 20 years.
- D. About 21 years.
- E. About 22 years.

3000

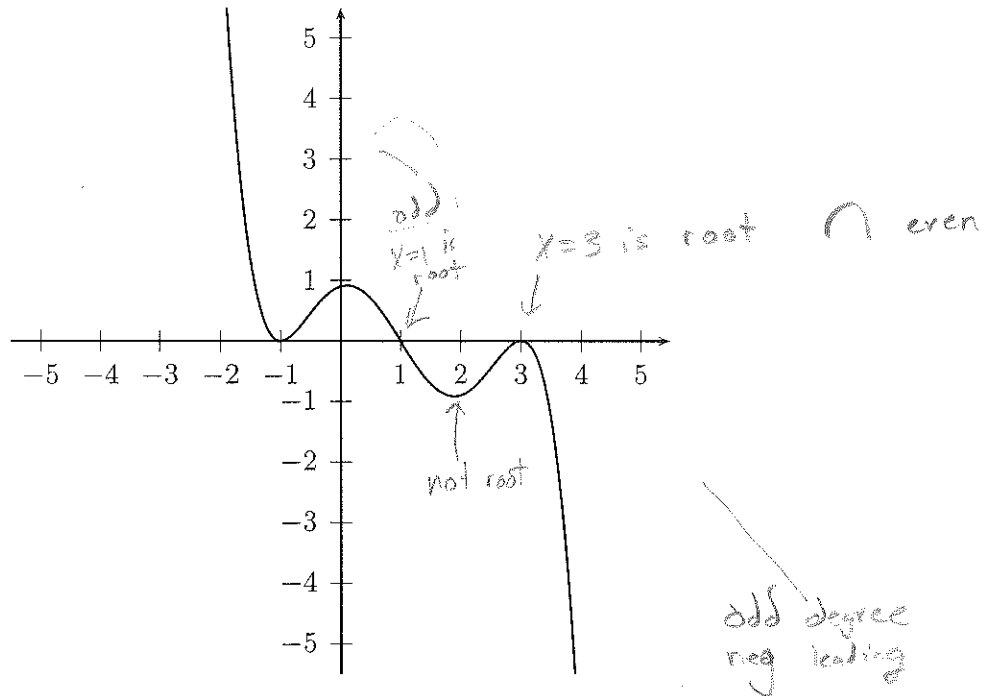
$$1000 e^{0.05t} = 3000$$

$$e^{0.05t} = 3$$

$$0.05t = \ln(3)$$

$$t = \ln(3) / 0.05 = 21.97 \text{ E}$$

The next two problems refer to the graph of the polynomial $P(x)$ shown in the picture below.



24. (5 points) For the graph of the polynomial $P(x)$ drawn below, which of the following can be concluded about $P(x)$?

- A. The degree of the polynomial is odd and the leading coefficient is positive.
- B. The degree of the polynomial is odd and the leading coefficient is negative.
- C. The degree of the polynomial is even and the leading coefficient is positive.
- D. The degree of the polynomial is even and the leading coefficient is negative.
- E. The parity (even or odd) of the polynomial and the sign of the leading coefficient cannot be determined from the graph.

25. (5 points) For the graph of the polynomial $P(x)$ drawn above, which of the following can be concluded?

- (I) $x = 3$ is a root of $P(x)$ with even multiplicity. YES
- (II) When $P(x)$ is divided by $x - 1$ the remainder is 0. YES
- (III) $x - 2$ is a factor of $P(x)$. NO rem is -1

- A. Statements (I), (II), and (III) are all true.
- B. Only statement (I) is true.
- C. Only statement (II) is true.
- D. Only statements (I) and (II) are true.
- E. Only statements (II) and (III) are true.

Formulae

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt} \quad (\text{if compounded } n \text{ times per year})$$

$$P(t) = P_0 e^{rt} \quad (\text{if compounded continuously})$$

Change of Base Formula

If $a, b, x > 0$ and neither a nor b equals 1, then

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Scrap paper

