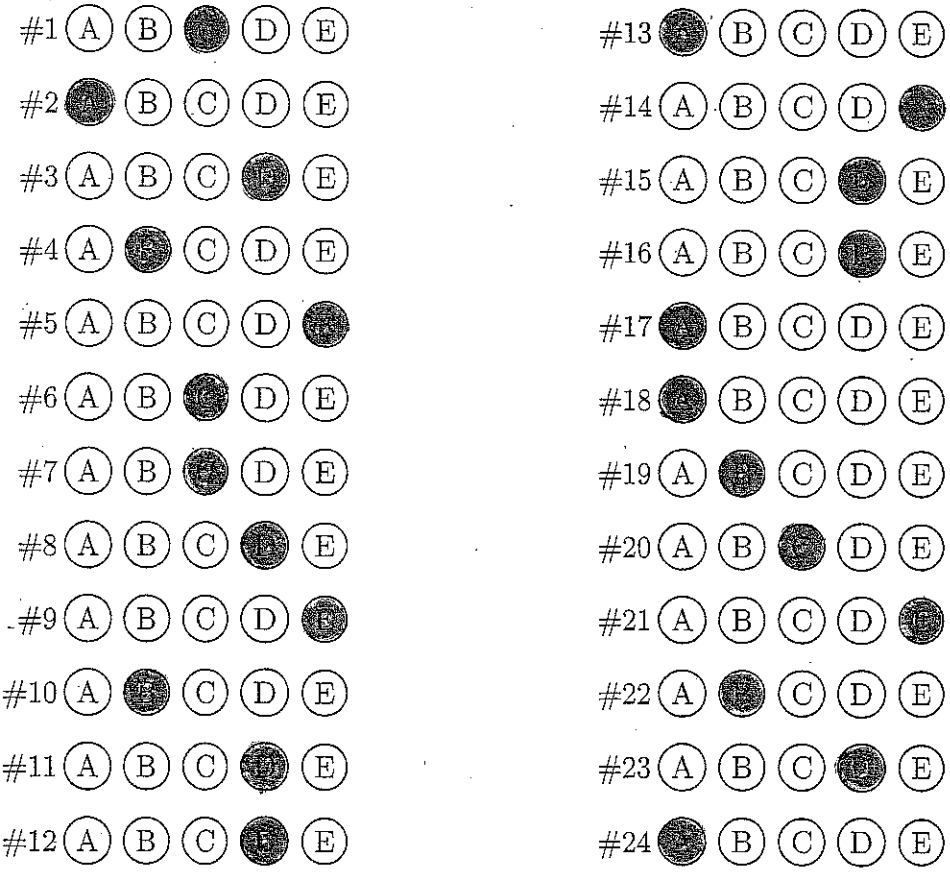
Directions:

- This is a two hour exam. Clearly print your name on the first page and the top of the third page (second piece of paper). No books, notes, internet connection, or cell phone can be used during this exam. Any scratch paper must be provided to you by the proctor and turned in with the exam. A calculator maybe used; however, the calculator cannot have a Computer Algebra System (CAS) or a QWERTY keyboard. When you have completed the exam:
  1) Turn in the entire exam (including cover page, and any scratch papers) to the proctor
  2) Show your ID to the proctor
  3) Sign the "Sign Out Sheet"
- All answers must be fully filled in on the front page, like so:

- The exam is out of 100 total points and includes 24 questions. Each question is 5 points and there are 4 extra credit questions. Only this front page will be graded and no partial credit will be awarded. Consequently, please double check to make sure that you have marked the answer you desire. A list of useful formulae and scrap paper is provided on the last page of the exam. Good Luck!



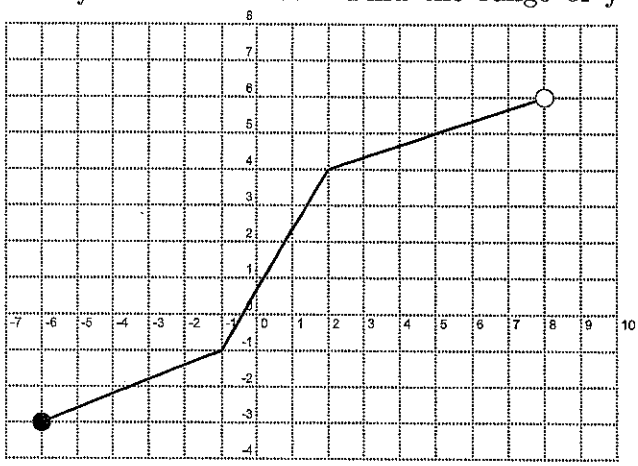
Name (Print): key

Section Number: _____

Section	Instructor	Class Start Time	Exam Location
001	Drew Butcher	MWF 8:00 AM	MEH
002	Konstantina Christodoulopoulou	MWF 9:00 AM	CB 106
003	Konstantina Christodoulopoulou	MWF 2:00 PM	CB 118
004	Drew Butcher	MWF 11:00 AM	MEH
005	Drew Butcher	MWF 3:00 PM	MEH (A-K) & CP 320 (L-Z)
006	Jonathan Constable	TR 8:00 AM	CB 102
007	Stephen Deterding	TR 8:00 AM	CB 110
008	Jonathan Constable	TR 9:30 AM	CB 102
009	Stephen Deterding	TR 9:30 AM	CB 110
010	Michael Gustin	TR 11:00 AM	CB 114
011	Robert Davis	TR 11:00 AM	CB 122
012	Michael Gustin	TR 12:30 PM	CB 114
013	Robert Davis	TR 12:30 PM	CB 122
014	Ray Kremer	TR 2:00 PM	CP 139
015	Clinton Hines	TR 2:00 PM	CP 139
016	Ray Kremer	TR 3:30 PM	CP 139
017	Clinton Hines	TR 3:30 PM	CP 139

UK: "Go CATS" Name: Key Version A Section:

1. (5 points) The graph of a one-to-one function f is shown below. Find the range of f^{-1}.



A. $(-6, 8]$
B. $(-3, 6]$
C. $[-6, 8)$
D. $[-3, 6)$
E. None of the above.

2. (5 points) Let $f(x) = \sqrt[3]{4-5x}$. Find $f^{-1}(x)$.

A. $f^{-1}(x) = \dfrac{4-x^3}{5}$
B. $f^{-1}(x) = \left(\dfrac{4-x}{5}\right)^3$
C. $f^{-1}(x) = \dfrac{x^3-4}{5}$
D. $f^{-1}(x) = 20 - x^3$
E. None of the above.

$y = \sqrt[3]{4-5x}$
$y^3 = 4-5x$
$y^3 - 4 = -5x$
$\frac{4-y^3}{5} = x$
$y = \frac{4-x^3}{5}$
$f^{-1}(x) = \frac{4-x^3}{5}$

3. (5 points) Let $f(x) = 4x^3 + 4x^2 - x - 1$. Determine the average rate of change of the function from $x = -1$ to $x = 1$.

A. 6
B. -6
C. $\frac{1}{2}$
D. 3
E. 0

$\frac{f(1)-f(-1)}{1-(-1)} = \frac{4+4-1-1-(-4+4+1-1)}{2} = \frac{8-2}{2} = \frac{6}{2} = 3$

$4+4-1-1=$
$4x^3+4x^2-x-1$
$-4x+4x+x-x$

4. (5 points) Determine the average rate of change of $f(x) = 5x^2 + 3$ from $x = a$ to $x = a + h$.

   A. $\dfrac{5h^2 - 3}{h}$

   B. $10a + 5h$

   C. 0

   D. $5a^2 + 3$

   E. 1

$\frac{f(a+h)-f(a)}{h} = \frac{5(a+h)^2+3-5a^2-3}{h} = \frac{5a^2+2ah+h^2)-5a^2}{h}$

$= \frac{5a^2+10ah+5h^2-5a^2}{h} = 10a+5h.$

5. (5 points) Suppose that the graph of f contains the point $(6, 4)$. Find a point that must be on the graph of $g(x) = f(5x) + 3$

   A. $\left(\dfrac{5}{6}, 7\right)$

   B. $(30, 7)$

   C. $(30, 4)$

   D. $\left(\dfrac{6}{5}, 4\right)$

   E. $\left(\dfrac{6}{5}, 7\right)$

$(\frac{6}{5}, 7)$. $5x=6$, $x=\frac{6}{5}$

6. (5 points) Explain how the graph of $g(x) = (x+2)^2+7$ is obtained from the graph of $f(x) = x^2$.

   A. Shift left 7 units and shift up 2 units.

   B. Shift right 2 units and shift up 7 units

   C. Shift left 2 units and shift up 7 units

   D. Shift left 2 units and shift down 7 units

   E. Shift right 2 units and shift down 7 units

shift left 2, shift up 7.

7. (5 points) Which of the given logarithmic statements is equivalent to the exponential statement.

$$5^3 = 125.$$

   A. $\log_3(125) = 5$

   B. $\log_5(3) = 125$

   C. $\log_5(125) = 3$

   D. $\log 125 = 3$

   E. None of the above.

$\log_5(125) = 3$

8. (5 points) Jon invests an initial deposit of \$420 dollars for 21 years in a savings account that has an interest rate of 8.442% compounded continuously. How much money is in the account after 21 years? Round your answer to the nearest cent.

$420 e^{0.08442 \cdot 21} \approx 2472.72$

A. \$2303.55
B. $\$4.13 \times 10^{79}$
C. \$3141.59
(D.) \$2472.72
E. \$1164.58

9. (5 points) The population of triffids in England is modelled by

$$p(t) = \frac{2000}{1 + 9e^{0.0625t}},$$

where t is measured in years since 1951. The population of triffids in 1951 and the population in 2001 was approximately:

$p(0) = \frac{2000}{1+9} = \frac{2000}{10} = 200$

$p(50) = \frac{2000}{1+9e^{0.0625 \cdot 50}} = 9.79$

A. In 1951, 2000 triffids. In 2001, 10 triffids.
B. In 1951, 2000 triffids. In 2001, 11 triffids.
C. In 1951, 1433 triffids. In 2001, 200 triffids.
D. In 1951, 189 triffids. In 2001, 11 triffids.
(E.) In 1951, 200 triffids. In 2001, 10 triffids.

10. (5 points) A culture has 50 bacteria at noon. If the number of bacteria triples every hour, then how many bacteria are in the culture at 5pm?

$50 \cdot 3^5$

A. 243
(B.) 12150
C. 250
D. 81
E. 4050

11. (5 points) The equation

$$P(t) = 35 \cdot 2^t$$

gives the number of bacteria in a culture after t hours. Which of the following statements is true?

A. The number of bacteria **decays** exponentially beginning with 2 bacteria.
B. The number of bacteria **decays** exponentially beginning with 35 bacteria.
C. The number of bacteria **grows** exponentially beginning with 2 bacteria.
(D.) The number of bacteria **grows** exponentially beginning with 35 bacteria.
E. None of the above

12. (5 points) Let f be the following one-to-one function

x	$f(x)$
1	3
2	4
3	2
4	1

Determine $f^{-1}(1)$.

$f^{-1}(1) = 4$.

A. 1

B. 2

C. 3

D. 4

E. None of the above

13. (5 points) Find the inverse of the one-to-one function

$$f(x) = 5x - \pi$$

$y = 5x - \pi$

$y + \pi = 5x$

$x + \pi = 5y$

$y = \frac{x+\pi}{5}$ $\quad f^{-1}(x) = \frac{x+\pi}{5}$

A. $f^{-1}(x) = \dfrac{x+\pi}{5}$

B. $f^{-1}(x) = \dfrac{x+5}{\pi}$

C. $f^{-1}(x) = \dfrac{x-\pi}{5}$

D. $f^{-1}(x) = \dfrac{x-5}{\pi}$

E. None of the above

14. (5 points) Let $f(x) = \dfrac{1}{5x+2}$ and $g(x) = x^2 - 2$. Find $f(g(x))$.

$f(g(x)) = \dfrac{1}{5(x^2-2)+2} = \dfrac{1}{5x^2-10+2} = \dfrac{1}{5x^2-8}$.

A. $\dfrac{1}{(5x^2-8)^2-2}$

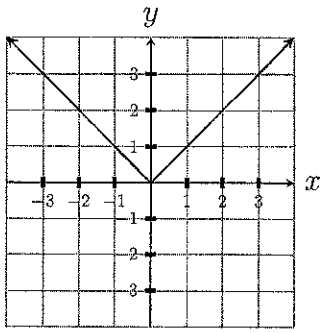
B. $\dfrac{1}{2x^2-4}$

C. $\dfrac{1}{2x^2}$

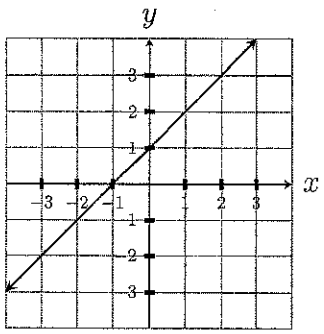
D. $\dfrac{5}{2x^2+8}$

E. $\dfrac{1}{5x^2-8}$

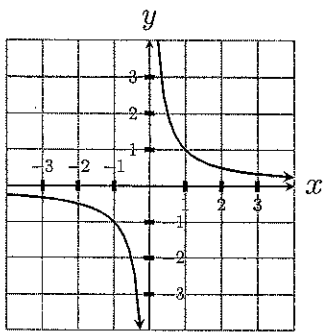
15. (5 points) Determine which of the following graphs represent a one-to-one function.



(I)



(II)



(III)

A. I and II

B. I, II, and III

C. III only

D. II and III

E. II only

16. (5 points) Find the domain of $\left(\frac{f}{g}\right)(x)$ where $f(x) = x^2 + x - 6$ and $g(x) = 2x + 5$.

A. $(-\infty, 2) \cup (2, \infty)$

B. $(-\infty, -2) \cup (-2, \infty)$

C. $(-\infty, 3) \cup (3, \infty)$

D. $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

E. $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

$g(x) \neq 0 \quad 2x+5 \neq 0 \quad 2x \neq -5 \quad x \neq -\frac{5}{2}.$

17. (5 points) Let $f(x) = x^2 + 3x$ and $g(x) = \sqrt{x-2}$. Find $g(f(x))$.

A. $\sqrt{x^2 + 3x - 2}$

B. $\sqrt{x^2 + x}$

C. $x^2 + 3x - 2$

D. $x - 2 + 3\sqrt{x-2}$

E. $(x-2)^2 + 3(x-2)$

$g(f(x)) = \sqrt{x^2+3x-2}$

18. (5 points) Given the following table of values, find $(f \circ g)(2)$.

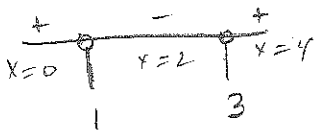
x	$f(x)$	$g(x)$
0	1	5
1	6	3
2	-2	1

$f(g(2)) = f(1) = 6$

(A) 6
B. 1
C. 5
D. -2
E. 3

19. (5 points) Find the domain of the function $f(x) = \ln(x^2 - 4x + 3)$.

$x^2 - 4x + 3 > 0$
$(x-1)(x-3) > 0$



A. $(-\infty, 1] \cup [3, \infty)$
(B.) $(-\infty, 1) \cup (3, \infty)$
C. $(1, 3)$
D. $[1, 3]$
E. None of the above.

20. (5 points) Write the given expression as a single logarithm:

$$3\log(5x) - 5[\log(x) - \log(y-8)]$$

$\log \frac{125x^3(y-8)^5}{x^5} = \log\left(\frac{125(y-8)^5}{x^2}\right)$

A. $\log(125x^2(y-8)^5)$
B. $\log\left(\frac{125}{x^2(y-8)^5}\right)$
(C) $\log\left(\frac{125(y-8)^5}{x^2}\right)$
D. $\log\left(\frac{15}{(y-8)^5x^4}\right)$
E. None of the above.

21. (5 points) Solve the equation

$$\log(5x+3) = \log(x+4) + \log(4)$$

$5x + 3 = 4(x+4)$
$5x + 3 = 4x + 16$
$x = 13.$

A. 9
B. $\frac{5}{4}$
C. -15
D. 3
(E) 13

22. (5 points) The number of bacteria in a culture is modeled by the function $P(t) = 200e^{0.5t}$ where t is measured in hours. Approximately how many hours will it take for the number of bacteria to reach 7600?

$200 \cdot e^{0.5t} = 7600$

$e^{0.5t} = 38$

$0.5t = \ln 38$

$t = \frac{\ln 38}{0.5} \approx 7.28.$

A. 4.33 hours

B. 7.28 hours.

C. 15.33 hours

D. 9.35 hours

E. None of the above.

23. (5 points) Solve the equation $3e^{x-5} = 7$ for x.

$e^{x-5} = \frac{7}{3}$

$x - 5 = \ln(7/3)$

$x = \ln(7/3) + 5$

A. $\ln\left(\frac{7}{3}\right) - 5$

B. $\frac{1}{3}(\ln(7) + 5)$

C. $\ln(4) + 5$

D. $\ln\left(\frac{7}{3}\right) + 5$

E. None of the above.

24. (5 points) Determine how much money will be in a savings account if the initial deposit was \$1,500 and the interest rate is 7% compounded quarterly for 5 years. Round your answer to the nearest cent.

$1500\left(1 + \frac{0.07}{4}\right)^{4 \cdot 5} = 2122.167\ldots$

A. 2122.17

B. 2103.83

C. 2752.93

D. 2126.44

E. None of the above.

Formulae

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0 e^{rt} \quad \text{(if compounded continuously)}$$

Change of Base Formula
If $a, b, x > 0$ and neither a nor b equals 1, then

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.$$

Scrap paper